Volume XVIII. *June, 1904.* *Number 6.*

THE

PHYSICAL REVIEW.

THE REFLECTING POWER OF SELENIUM AS DETERMINED BY A SPECTROPHOTOMETER.

By C. K. Edmunds.

THE present photometric measurements of the reflecting power of selenium are confined to the visible region of the spectrum, and are divided into two main parts. (1) Determination of the reflecting power of selenium for several different wave-lengths for normal incidence. (2) Determination of the reflecting power for three different wave-lengths of light polarized in and normal to the plane of incidence, for a range of incidences from 20°–80°. The results under (1) are compared with the reflection curve secured by calculation from the dispersion and absorption curves; those under (2) are compared with the values calculated by means of Cauchy's formulæ, assuming the values of the absorption coefficient and refractive index secured from the metallic reflection experiments.

Apparatus and Method of Observation.

The spectro-photometer used was made by Schmidt and Haensch after Professor Brace's[1] design. It consists essentially (Figs. 1 and 2) of a double prism P with a narrow silvered strip SS on the face AD of the right half. Light of the same wave-length is thus brought by direct transmission from the collimator T, and after reflection at the silver strip from T', to the same focus in the observing telescope R. When the eyepiece is removed and the prism viewed through a slit in the focal plane of R, the eye sees three

[1] Brace, Phil. Mag., 48, 420–430, 1899, and Astro. Phys. Jour., 11, 6–24, 1900.

fields (Fig. 3), the central one $ABCD$ illuminated by light from the right collimator T', and the upper and lower ones ACF and BDG, by light from the left collimator T. The fields meet in the sharp edge of the silver strip. A match in intensity is secured by altering the width of T' which must be bilateral in order that the spectra from the two collimators may be kept in-step, after being once adjusted, as the slit T' is changed in width. The condition of photometric equality is determined by the vanishing of the line of separation between the two portions of the field of view illuminated

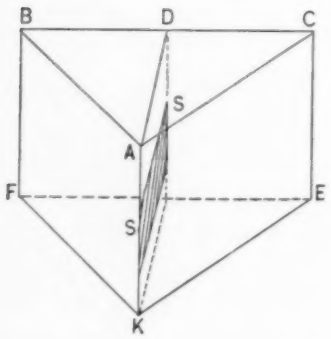

Fig. 1.
Double Prism, Silver Strip: ss.

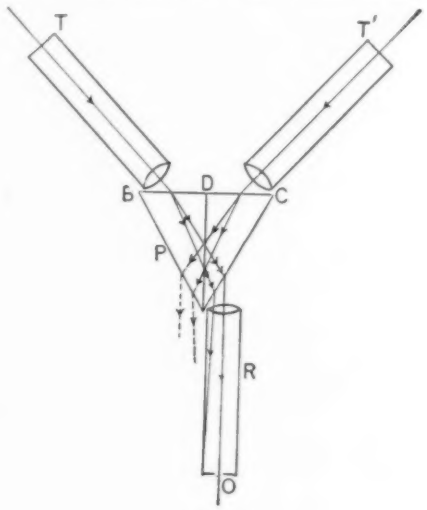

Fig. 2.
Optical System.

respectively by the direct and the reflected light. By shifting the telescope R, different portions of the spectrum may be brought into the field of view and so the intensities of the various colors of the reflected beam may be compared with the corresponding colors of the direct light, and the reflecting power for any desired wave-length determined.

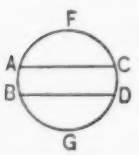

Fig. 3.
Field of View.

The conditions of an ideal photometer screen as given by Lummer and Brodhun[1] are met by this instrument in a manner described by Tuckerman,[2] who has so fully explained the adjustments necessary to their fulfillment that no additional account need be given here.

[1] Lummer and Brodhun, Zeitschrift für Instrumentenkunde, 12, 137–138, 1892.
[2] Tuckerman, Astro. Phys. Jour., 147, 149, Oct., 1902.

General Method of Observation.

The slit of one collimator T is, after the initial adjustments of the photometer, kept fixed in width throughout a whole series of comparisons. The slit on the other collimator T' is controlled by an accurately turned screw by means of which its width can be altered at will by a known amount, and thus the total amount of light passing down this collimator can be changed as desired. The amount of light coming down the collimator with the fixed slit can be changed without altering the width of the slit by means of a rotating disk mounted so as to rotate directly in front of the fixed slit and to cut off the light during a certain fraction of each revolution. The reflecting power of the given mirror is determined in the following manner. Light from the same source or two sources practically identical, is in part sent down the collimator with the fixed slit and in part sent down the collimator with the adjustable slit the width of which is altered till a photometric equality of the two beams is secured in the field of view. Call the width of the adjustable slit under these conditions S'.

The mirror to be investigated is now introduced in the path of the light on the way to the adjustable slit and the direction of the incident beam and the portion of the mirror altered till after reflection at the desired incidence the light again enters the adjustable slit and so into the photometric system. Photometric equality will no longer exist in the field of view, since the reflected beam has been weakened in intensity by the reflection, and a match can be secured by widening the adjustable slit. Call its new width S. Now if the direct ratio-law held (*i. e.*, if the total amount of light passing throught a slit were directly proportional to the slit width) the ratio of the intensity of the reflected light to that of the incident light, *i. e.*, the reflecting power of the mirror for the color matched and for the angle of incidence used would be S'/S, the intensity of a beam being inversely proportional to the width of the slit required to secure a match between it and a fixed illumination.

As a matter of fact, this direct ratio-law does not hold in general and the optical value of different slit widths must be determined for different colors and different intensities of radiation in a manner de-

scribed[1] elsewhere by Capps and others. By means of calibration curves secured in the manner there described, this ratio-by-slit, *i. e.*, S'/S can be reduced to give the correct value of the reflecting power.

If the reflected light is very much weaker than the direct, the change in width of the adjustable slit required for a match may be too great either for the limited range of the adjusting screw or for employing the best portion of the calibration curve. In this case the amount of light coming down the collimator with the fixed slit may be decreased without altering the slit width, by cutting down the light falling on it by means of a rotating sector of known and adjustable angular value revolving directly in front of the fixed slit in such a way as to cut off the light during a certain fraction of each revolution. The speed at which this sector is run does not enter except that it must be high enough not to cause any flickering in the field of view.

If now, the mirror being removed and the sector at rest so as to allow full illumination of both the fixed and adjustable slits, the width of the latter required for a match be S', the mirror is put in position, the reflected beam sent upon the adjustable slit, the rotating sector made to revolve and its angle changed till a match is again nearly secured in the field of view. An exact match can now be obtained by a slight change in the width of the adjustable slit. As before call its new width S. If the angle of the sector used was such as to allow light to pass during $1/n$th of a revolution, then the reflecting power is $S'/S \cdot 1/n$. If the change in the width of the adjustable slit is not small, then the value of S'/S must first be optically calibrated by means of the curves showing the variations from the direct ratio-law, and then the reduction factor $1/n$ applied to give the final value of the reflection power for the color matched and the angle of incidence used.

CALIBRATION OF SPECTROPHOTOMETER.

Owing to the curvature of the luminosity-curve of the radiant the illumination is not in general proportional to the slit width. With a bilateral slit the direct ratio holds at points where the curve is a

[1] Murphy, Astro. Phys Jour., 6, 1–10, 1897.
Brace, Phil. Mag., 48, p. 425, 1899.
Capps, Astro. Phys. Jour , 11, 25–35, 1900.

straight line, but where the luminosity-curve is convex to the axis of wave-lengths, the intensity of the light passing down the collimator T' for different widths of slit increases in a greater ratio than the width of the slit, the reverse being true for the concave portions. Hence the discrepancies, which are greater the greater the width of the fixed slit T, are in an opposite sense in the yellow from those in the red and blue; slit readings give too high a ratio for red and blue light, too low for yellow.

In the present experiments the bilateral slit T' was calibrated optically by means of a rotating sector, for the width of T (.3 mm. in most cases) and the various wave-lengths used, in the manner devised by Professor Brace and employed by Capps and others. A simple cardboard disk mounted on a small motor, with its circumference divided into a number of parts and slotted out to different depths between different radii as shown in Fig. 4, was rotated between the source and the slit T, so that the different sectors covered the slit in its successive positions, and thus the intensity of the light coming down the collimator T was varied in known ratios depending on the angle of the sector used. A twelve part disk was used, with a greatest diameter of 24 cm. Since Capps[1] has recently fully described the method and results of similar calibrations further detail will not be given here, other than to point out that the final readings were made by a combined use of the variable slit and the rotating sector. Approximate matches were secured by cutting down the light entering T by appropriate sectors of known angular value and only the final adjustment for a match was made by varying the width of slit T', the effort being to confine the readings to as nearly the same region on the screw operating T' as possible. Whenever the variation was sufficiently great the ratio by slit readings was corrected by means of the calibration curves and the reduction factor of the disk applied to this corrected ratio. This was the method first used by Brace and More[2] in 1899, it eliminates to a great extent any errors of the screw, giving greater accuracy in the optical evaluation of the slit comparisons.

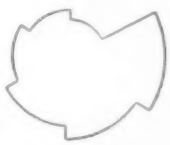

Fig. 4.
Calibratory Disk.

[1] Capps, Astro. Phys. Jour., 11, 25–35, 1900.

[2] Brace, Bull. Am. Phys. Soc., 1, 2, pp. 29–31; also Brodhun, Zeitschrift für Instrumentenkunde, 20, 210–212, 1900.

With narrow slit width, such as here used, the zero correction for T' becomes an important factor. From time to time between various sets of observations the bilateral slit T' was subjected to a special test, and by means of a micrometer microscope its width for different readings of the screw's divided head were noted in the region of small slit widths, and from them the true zero was determined by extrapolation. After starting to use the slit, it was never again closed without the zero being redetermined. Final settings for photometric equality were always made with increasing slit-width.

AUXILIARY REFLECTING SPECTROMETER.

The general arrangement of the apparatus is shown in Fig 5. S is the Brace's spectrophotometer. S' is the auxiliary reflecting spectrometer carrying the mirror M mounted on the graduated table V in the center of the larger table D, about which the collimator T'' can be rotated for any desired angle of incidence. L and

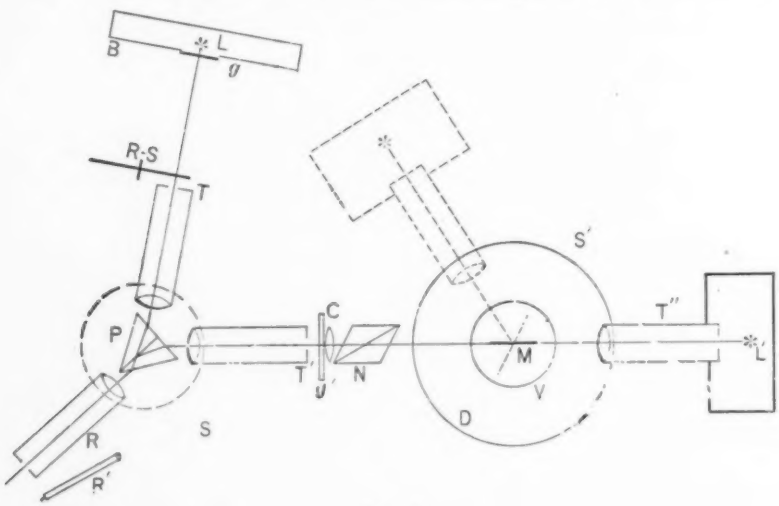

Fig. 5.

Assembly of Photometric Apparatus. Dimensions: From center of P to T, or to T', approximately, 30 cm. Diameter $V = 13$ cm., diam. $D = 40$ cm., from T to $g = 40$ cm. Focal length of T'', approx., 35 cms., aperature 4 cm. in diam. From center of P to M, approx., 80 cm.

L' are the 110-volt Nernst filaments enclosed in the boxes B and B'. g is a window of thin milk glass, and g' is a screen of fine ground glass. N is a Nicol's prism of 3 cm. aperture, by means of which

the reflected light can be polarized in any desired azimuth, the zero position of the Nicol being determined in a way described presently. C is a condensing lens which focuses the reflected light on the bilateral slit T'. $R - S$ is the rotating divided disk mounted on a motor carried by a lathe tool-carriage so as to shift the disk in its own plane back and forth before T.

The radiant L was mounted on an extension rigidly connected with the platform carrying the spectrophotometer S. The motor carrying the rotating sector was mounted on a heavy stone sill entirely separate from the other mountings so as to cause no vibration. The ground glass g', the lens C and the Nicol N were all mounted on the base of S. The radiant L' was rigidly attached to the collimator arm of T''. The various dimensions were as shown in the diagram, approximately.

Light from L passed through the milk glass g, was then interrupted by the rotating sector $R - S$ entered through the slit T, was rendered parallel by this collimator, suffered refraction and dispersion in P and entered the telescope R where it was viewed through the ocular slit O. The slit T was kept at a fixed width and the intensity of the field from T altered by shifting the sector $R - S$.

Light from L' was rendered parallel by the collimator T'', fell on the mirror M at any desired incidence, (the value of which was given by the circle reading on V, which was graduated to $1'$) and after reflection was polarized in any desired azimuth by passing through N, was focused by C on the bilateral slit T' after passage through the ground glass screen g', which helped to give a uniform field, was again rendered parallel by the collimator T' and suffered refraction, reflection and dispersion in P, the part striking the silver strip SS (Fig. 1) entering the viewing telescope R and forming the central part of the field as viewed through the ocular slit O. The part of the light from T' which passed above and below the silver strip SS was refracted out and lost in the box surrounding P. The intensity of this central portion of the field was changed by altering the width of T' by means of a micrometer screw the setting of which could be read off by means of a small auxiliary telescope R' mounted along side of R.

The smallest angle of incidence at which observations could be

secured was 7°, and the values recorded for normal incidence are really for $\varphi = 7° \pm$. It is not thought necessary, however, to make any allowance for this small angle.

In the following tables the recorded values for slit width of 7′ are the means of ten settings in each case. The variation in such a set of ten readings is exhibited by the sample given in Table I. The maximum deviation from the mean is 1.5 per cent. and the probable error of the mean value is .4 per cent. The probable error in the final values of the reflection power is about .75 per cent., though it is very likely greater than this for the extreme blue, on account of the difficulty of good seeing in that region.

Comparison of the Values of the Reflecting Power as Secured by Different Methods.

There are in general two ways of determining the reflecting power of a given substance, (1) direct and (2) indirect. (1) The direct method is by such photometric measurements as have just been described. (2) The indirect method involves the use of the formula

$$R = \frac{(n-1)^2 + k'^2}{(n+1) + k'^2}, \qquad (a)^1$$

where

n = refractive index

and

k' = absorption coefficient,

at normal incidence, for the wave-length considered.

These optical constants n and k' can be determined either (a) directly by measurements with prisms and thin films or (b) indirectly by calculation from the constants of elliptical polarization as described in an earlier paper.

The values of the reflecting power for both old and new mirror are, together with those secured from the experiments of others presented in Table III., the first column of which gives the wave-length of the incident light, the second the reflecting power as I have calculated it from Pfund's observations (see page 395) of the absorption and refraction indices by means of equation (a). Column 3 gives the values of the reflecting power as calculated by the same

[1] Cauchy, Compt. rend., T. VIII., 1839, p. 560.

Observations.

Table I.

Vitreous Selenium. Mirror 5 months old. Direct light. Date: March 25, 1903.

Telescope.	5.58.	6.08.	6.33.	6.83.	7.08.	7.58.	7.83.
Wave-length.	489 μμ.	524.	545.	589.	622.	701.	748 μμ.
	Divs						
Slit	124	128	136	129	147	190	212
	3	24	5	30	44	85	2
	3	27	3	30	43	91	4
	3	27	8	29	43	92	4
	2	29	5	29	43	88	2
	3	26	8	31	43	89	2
	2	24	5	32	47	89	3
	3	27	6	28	47	90	2
	4	26	7	31	43	87	4
	123	128	5	132	145	189	213
Means	123.0	125.6	135.8	130.1	144.5	189.0	212.8
Zero correc.	−19.8	−19.8	−19.8	−19.8	−19.8	−19.8	−19.8
Corrected mean	103.2	105.8	116.0	110.3	124.7	169.2	193.0
Maximum deviation	.1	2.4	2.8	2.1	2.5	3.0	1.2
Or	1 %					1.5 %	
Probable error in mean	.04					.4	
Or	.04%					.4 %	

Reflecting Power of a Mirror of Vitreous Selenium.

Table II.

Normal Incidence.

Surface 5 months old.

Wave-length.	489 μμ	524	545	589	622	701	748 μμ
	Corrected Slit Readings.						
Reflected: S	143.35	125.60	167.25	174.65	159.91	148.41	144.4
Direct: S'	103.2	105.8	116.0	110.3	124.7	169.2	193.0
Reduction factor of disk: $1/n$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{8}$
$S' \cdot 1/n$	34.4	35.266	38.66	36.766	41.566	28.2	32.166
R in %	24.01	28.08	23.12	21.05	25.99	19.00	22.28

Fresh Mirror (i. e., a day or two old).

Wave-length.	432 μμ	454	489	524	589	622	660	748 μμ
S	207.2	245.6	274.4	265.6	281.7	287.2	307.1	432.4
S'	192.0	197.9	204.0	209.3	218.0	746.5	334.4	381.5
$1/n$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{4}$
$S' \cdot 1/n$	64.0	65.97	68.0	69.77	72.67	82.17	83.6	95.38
R in %	30.89	26.86	24.78	26.27	25.80	28.61	27.22	22.06

formula from the values of the optical constants deduced from the metallic reflection experiments of various observers by application of Walter's formulæ.

TABLE III.

Reflecting Power of Vitreous Selenium.

1	2	3	4	5	
	Calculated from Pfund's Observations.	Calculated from Metallic Reflection Experiments.	Directly Observed by Nutting.	Directly Observed by Edmunds.	
214			.0998	New (*a*)	Old (*b*)
280		.2008			
334			.1265		
395			.1584		
400	.2703				
410			.1667		
415	.2713				
425	.2687				
431		.2679	.1583		
432		.2710		.3089	
439		.2534			
442	.2692				
454				.2686	
466	.2724				
486		.2723			
489				.2478	.2401
490	.2745				
508		.2590			
510			.1685		
515	.2662				
524				.2627	.2808
527		.2706			
545					.2312
550	.2555 Quincke		.1801		
589	.2417 (.2722)	.2600	.1834	.2580	.2105
605			.1763		
620			.1434		
622				.2861	25.99
631		.2174			
640	.2202		.2194		
656		.2609			
660				.2722	
701					19.00
710	.2052				
721		.2101			
735		.2279			
748				.2206	22.28
760	.1989				

Column 4 presents Nutting's[1] photometrically determined value of the reflecting power, R.

In column 5 are recorded my photometric values of R for both old (b) and new (a) mirror.

These results are represented graphically in Fig. 6.

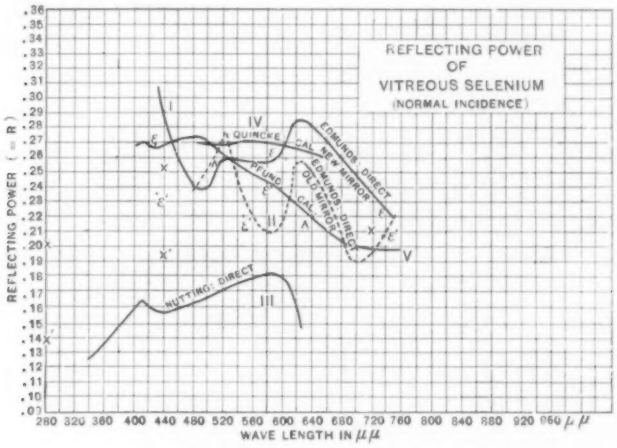

Fig. 6.

Reflecting Power of Vitreous Selenium.

Curve I., photometrically observed for a fresh mirror by Edmunds; II., photometrically observed for a mirror five months old by Edmunds; III., photometrically observed by Nutting, probably for an old mirror; IV., calculated (Edmunds) from Quincke's observation on metallic reflection; V., calculated (Edmunds) from Pfund's direct observations on refraction and absorption. Isolated points have been calculated from metallic reflection experiments of various observers, as follows: Edmunds, new mirror ε; Edmunds, old mirror, ε'; Cornu, new mirror x, old mirror x'; Glan, $\wedge$.

The Surface Color of Selenium.

The directly determined reflection curve for selenium at normal incidence (see Fig. 6) shows a maximum at about 620 and a minimum at about 490 $\mu\mu$ after which there is a steady rise in the reflecting power as the wave-length is decreased.

The high values of the absorption in the ultra-violet as found by Pfund suggest possible selective reflection in this region. But, as already noted, several attempts to detect "Reststrahlen" in the ultra-violet by successive multiple reflections from six mirrors of vitreous selenium failed to show any selective reflection; in fact,

[1] Nutting, Phys. Zeit., 201–206, Jan. 1, 1903.

they indicated a slight falling off in reflecting power. This agrees with the drop in Nutting's photometrically observed reflection curve, which occurs at about $\lambda = 410\,\mu\mu$ from which point on (*i. e.*, for decreasing wave-lengths) there is a rapid falling off in reflecting power (see Fig. 6).

The method followed in looking for selective reflection in the ultra-violet was to allow the light from a cadmium spark to suffer six-fold-reflection at nearly normal incidence from six good mirror surfaces of vitreous selenium, and after the sixth reflection to fall on the slit of a quartz spectrograph in which the light was received upon a photographic plate and an exposure made for a certain known time-interval. Several exposures of different intervals were made on the same plate. The mirrors were then removed and the light from the spark allowed to fall directly on the slit of the quartz spectrograph. Several exposures of different time intervals were again made on the same plate used for the reflected light. Comparisons of the relative density of the developed plate for different wave-lengths showed no decided increase in the reflecting power for wave-lengths from 345 to $229\,\mu\mu$.

The relation of reflection and absorption is well illustrated by the curves for selenium. Metallic reflection experiments show a slight maximum in the absorption at about $590\,\mu\mu$ and then a steady rise for shorter wave-lengths. The photometrically determined reflection curves show a maximum in reflecting power at about $620\,\mu\mu$ and $420\,\mu\mu$ which in each case lie on the side of larger wave-lengths from the maxima in the absorption curve, that is, the maximum of the reflection curve is shifted in the direction of the longer wave-lengths with respect to the absorption maximum, while the minimum of the reflection curve lies on the blue side of the absorption-band, a result in full accord with the deductions from formula (*a*).

Reflecting Power of Vitreous Selenium for Light Polarized in and Normal to the Plane of Incidence, at Different Incidences and for Different Colors.

By means of the arrangement of apparatus as pictured in Fig. 5, the light could be received by the selenium mirror M at any desired angle and after reflection could be polarized in any desired

azimuth by means of the Nicol N, the zero position of which was determined thus:

The polarizing angle of a flint glass prism for sodium light was calculated from $\tan \varphi_p = n$, and $n_D = 1.5183$. The prism was mounted on V in place of the mirror M and sodium light reflected from one of its surfaces at this polarizing angle 56° 38′. N was then turned in its mounting until the light reaching T' was a minimum. In this position the polarizing plane of the Nicol was normal to the plane of incidence, and light passing through it was polarized normal to the plane of incidence.

The observations of the reflecting power for light polarized normal to the plane of incidence and for light polarized in the plane of incidence for incidences of 20°, 40° are reproduced as examples, and the final summary (R and R') compares the observed values of R for incidences 20°–80° with the values computed from the determinations of the refractive index and absorption coefficient previously recounted.

In Tables R and R' the columns headed d give the difference between the observed and calculated values of the reflecting power ($d \equiv$ calculated — observed).

The agreement between observed and calculated values is on the whole fair, being somewhat closer in the case of light polarized in the plane of incidence than for light polarized normally. In the first case the discrepancy is much greater for the larger angles of incidence, probably on account of the greater influence of the surface film at such incidences. In the second case, *i. e.*, of light polarized normal to the plane of incidence, the discrepancy between observed and calculated reflecting powers is greater for the small incidences.

For blue light the values calculated from Pfund's direct measures of refraction and absorption agree very well with the observed reflecting power for light polarized in the incidence plane; though for light polarized normally the agreement is not so good.

For yellow light the values calculated from my metallic reflection experiments agree very well (better than those calculated from other data) with the directly observed values of the reflecting power; though as in all other cases also the highest angles of incidence must be excepted.

No calculated values are recorded for the red ($\lambda = 690\ \mu\mu$) because no sufficiently reliable determinations of the refractive index and absorption coefficient have been made for wave-lengths very near

Reflected Light Polarized in the Plane of Incidence.

	$\phi = 20°$.		
Telescope	7.°50	6.°83	5.°75
$\lambda =$	690 $\mu\mu$	589	501
Disk	.2500	.2500	.3333
S	175.2	172.7	218.4
S'	172.0	175.7	187.1
S'/S	.9875	1.0174	.8567
Calibrated	.9875	1.0174	.8020
Reduced K_i^p	.2469	.2544	.2673
	$\phi = 40°$.		
Disk	.4167	.4167	.5833
S	158.0	149.7	196.0
S'	141.0	126.7	140.3
S'/S	.8924	.8464	.7158
Calibrated	.853	.821	658
Reduced K_i^p	.3554	.3421	.3838

REFLECTING POWER OF VITREOUS SELENIUM AT DIFFERENT ANGLES OF INCIDENCE.

Reflected Light Polarized Normal to the Plane of Incidence.

	$\phi = 20°$.		
Telescope at	7.°50	6.°83	5.°75
$\lambda =$	690 $\mu\mu$	589	501
Reflected: Disk	5/12	5/12 (.4167)	6/12
Slit: S	139.3	120.6	126.5
Direct: S'	68.2	72.0	58.3
S'/S	.4896	.5970	.4609
Calibrated	.466	.631	.420
K_i^n in per cent. reduced	19.42	26.29	21.00
	$\phi = 40°$.		
Disk	5/12	6/12 (.5000)	6/12
S	170.7	166.3	137.3
S'	68.2	72.0	58.3
S'/S	.3995	.2851	.4246
Calibrated	.346	.280	.382
K_i^n in per cent. reduced	14.42	14.00	19.10

TABLE R′.

Reflecting Power, Observed and Computed. Light Polarized in the Incidence Plane.

	$\lambda = 501\ \mu\mu$					$\lambda = 589\ \mu\mu$									$\lambda = 690\ \mu\mu$
		1	2				1	2	3	4					
ϕ	Observed.	Cal.	Cal.	δ_1	δ_2	Observed.	Cal.	Cal.	Cal.	Cal.	δ_1 °	δ_2	δ_3	$'\delta_4$	Observ d.
20°	.2673	.2752	.2914	+.0079	+.0241	.2544	.2813	.2554	.2624	.2764	+.0269	+.0010	+.0080	+.0220	.2469
40	.3838	.3534	.3880	−.0304	+.0042	.3421	.3543	.3278	.3337	.3491	+.0122	−.0143	−.0084	+.0070	.3554
60	.5000	.5055	.5156	+.0055	+.0156	.4871	.5064	.4822	.4864	.5016	+.0193	−.0049	.0007	+.0145	.4670
70	.6583	.6267	.6349	−.0316	−.0082	.6030	.6275	.6068	.6099	.6233	+.0245	+.0038	+.0069	+.0203	.6237
80	.7175	.7886	.7936	+.0711	+.0050	.7185	.7890	.7759	.7776	.7863	+.0705	+.0574	+.0591	+.0678	.7010

$\delta \equiv$ cal. — observed.

For $\lambda = 501\ \mu\mu$, col. 1 has been calculated from Glan's values of $\bar{\phi}$ and $\bar{\psi}$ for λ 508 $\mu\mu$ which give $n_0 = 2.884$, $k'_0 = .9033$.
" " " 2 " " " " Pfund's values of $n_0 = 3.11$, $k'_0 = .388$ for $\lambda = 501$.

For $\lambda = 589\ \mu\mu$, col. 1 has been calculated from Edmunds' values of $\bar{\varphi}$ and $\bar{\psi}$ for *new* mirror, giving $n_{\bar{\Phi}} = 2.8198$, $k'_{\bar{\Phi}} = 0.8073$.
" " " 4 " " " " the same reduced to give $n_0 = 2.8076$, $k'_0 = .7637$.
" " " 2 " " " " Edmunds' values of $\bar{\varphi}$ and $\bar{\psi}$ for *old* mirror, giving $n_0 = 2.406$, $k_0' = 0.9863$.
" " " 3 " " " " Wood's values of $n_0 = 2.925$, $k_0' = 0.155$.

TABLE R.

Reflecting Power, Observed and Computed. Light Polarized Normal to the Incidence Plane.

ϕ	$\lambda = 501\ \mu\mu$					$\lambda = 589\ \mu\mu$									$\lambda = 690$
		1	2				1	2	3	4					
	Observed.	Cal.	Cal.	δ_1	δ_2	Observed.	Cal.	Cal.	Cal.	Cal.	δ_1	δ_2	δ_3	δ_4	Observed.
20°	.2100	.2336	.2486	+.0236	+.0386	.2629	.2387	.1963	.2209	.2341	−.0242	−.0666	−.0429	−.0287	.1942
40	.1910	.1710	.1957	−.0200	+.0047	.1400	.1716	.1501	.1556	.1675	+.0316	+.0101	+.0156	+.0275	.1442
60	.0635	.0575	.0605	−.0140	−.0030	.0697	.0592	.0531	.0448	.0563	−.0105	−.0166	−.0249	−.0134	.0438
70	.0362	.0125	.0065	−.0237	−.0297	.0314	.0162	.0305	.0013	.0146	−.0152	−.0009	−.0301	−.0168	.0199
80	.1187	.0991	.0772	−.0196	−.0415	.1000	.1069	.1544	.0898	.1071	+.0069	+.0544	−.0102	+.0071	.0842

The calculated values are based on the same determinations of n_0 and k'_0 which are used for Table R′, just given. ($\delta \equiv$ cal. − observed.)

this. The calculations are somewhat tedious and were not thought justified if based on interpolated values of the optical constants.

The curves in Fig. 7 showing the reflecting power of selenium at various incidences and for the three colors, $\lambda = 501\ \mu\mu$, $589\ \mu\mu$ and $690\ \mu\mu$, indicate that at 70 degrees incidence there would be very little reflected light of any color polarized normal to the plane of incidence, while in the reflected light polarized in the plane of incidence there would be an excess of blue and red, and a minimum of yellow. This is an agreement with some previous observations made to detect any trace of surface color in the visible region of the spectrum. Ordinary light was reflected from vitreous selenium and

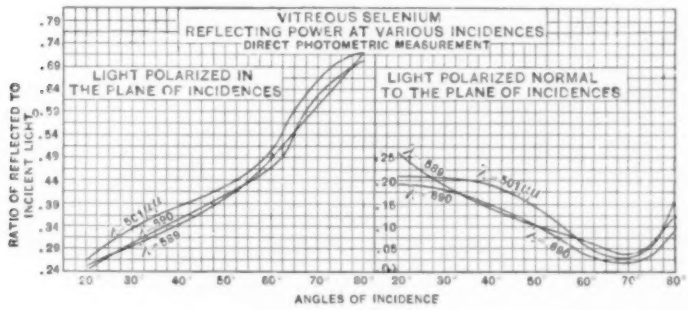
VITREOUS SELENIUM
REFLECTING POWER AT VARIOUS INCIDENCES
DIRECT PHOTOMETRIC MEASUREMENT
LIGHT POLARIZED IN THE PLANE OF INCIDENCES
LIGHT POLARIZED NORMAL TO THE PLANE OF INCIDENCES
RATIO OF REFLECTED TO INCIDENT LIGHT
ANGLES OF INCIDENCE

Fig. 7.

the reflected light viewed through a Nicol's prism. The angle of incidence was changed and the azimuth of the Nicol step by step, and it was found that at about 70° incidence, reflected light polarized in the plane of incidence showed a marked blue tint.

Murphy has given results for the intensities of polarized light reflected from the face of a Steinheil prism as determined by means of a Lummer-Brodhun spectral photometer. They show a close agreement between the observed and computed values, the differences being in every case but a small part of the total incident light. The greatest discrepancy exists for light polarized perpendicular to the plane of incidence, and for the small angles of incidence. The agreement between calculated and observed values of the reflecting power is nearly as good for selenium as it is for glass, especially if we take into account the great difficulty of controlling the surface condition of a selenium mirror as compared with a glass surface.

SYNOPSIS OF RESULTS.

The present photometric measurements of the reflecting power of selenium have shown that there is in general an increase in the reflecting power as the wave-length of the incident light is decreased, there being in the course of this general rise a maximum at about 620 $\mu\mu$ and a minimum at about 590 $\mu\mu$ after which for shorter wave-lengths there is a steady increase in reflecting power.

An apparent effect due to lapse of time, *i. e.*, ageing of the mirror in such a way as to form an effective surface layer, greatly modifies this curve. The reflecting power for all wave-lengths is decreased except in the region of 525 $\mu\mu$ where an increase was noted.

Nutting's observed values of the reflecting power are entirely below the field of other determinations and the present experiments would seem to indicate that his mirror had been made some time before using or at least had acquired a surface film similar to that on the mirror described as "old" in this paper.

The relation of reflection to refraction and absorption was treated briefly and the deductions from the reflection formula found to be verified in a general way.

Photometric measures verified Cauchy's formulæ giving the reflecting power of a metal (*i. e.*, a heavily absorbing medium) in terms of the optical constants for various angles of incidence. The agreement between observed and calculated values was nearly as satisfactory as for glass to which Fresnel's formulæ giving the reflecting power of a transparent substance in terms of the refractive index had been applied.

In conclusion, I desire to express my appreciation of the kindness of Professor Ames and of Professor Wood. The latter suggested the investigation to me, and both have given advice and liberal aid throughout the course of work.

PHYSICAL LABORATORY, JOHNS HOPKINS UNIVERSITY,
May, 1903.

STUDIES IN LUMINESCENCE. II.

By Edward L. Nichols and Ernest Merritt.

A Spectro-photometric Study of Fluorescent Solutions Belonging to Lommel's First Class.[1]

THE law expressed by Stokes[2] in his memoir entitled "The change of Refrangibility of Light" to the effect that in fluorescence the fluorescent light is always of greater wave-length than the exciting light has been called in question by Lommel, who pointed out that for certain fluorescent bodies there is an unmistakable overlapping of the regions in the spectrum occupied by the exciting light and by the fluorescence which it produces. Lommel made the further very important statement that for this class of substances the character and composition of the fluorescence spectrum is independent of the wave-length of the exciting light. Lommel's results in so far as they had to do with the non-validity of Stokes's law were confirmed by Hagenbach.[3] A few years later Lubarsch[4] published measurements in confirmation of Stokes's law. A later paper by Lommel,[5] in which he described the fluorescence of the so-called chameleon colors, led Hagenbach to new experiments, in the course of which he discovered what he believed to be a source of error in his former measurements, and he reaffirmed the law of Stokes for all such substances. In 1877 Brauner[6] obtained results in confirmation of Lommel's view. In 1879, Lubarsch[7] published further experiments on fluorescence, this time in favor of Lommel's results. Lamansky[8] in 1879 described meas-

[1] Read at the St. Louis meeting of American Physical Society, Dec. 29, 1903, under the title "The Spectrometric Study of Fluorescence."

[2] Stokes, Phil. Transactions 1852, p. 463.

[3] Hagenbach, Poggendorff's Annalen, 146, pp. 65, 232, 373, 508.

[4] Lubarsch, Poggendorff's Ann., 153, p. 428, 1874.

[5] Lommel, Poggendorff's Ann., 159, p. 514, 1876.

[6] Brauner, Wiener Anzeiger, 19. p. 178, 1877.

[7] Lubarsch, Wied. Ann., 6, p. 248.

[8] Lamansky, Comptes Rendus, 88, p. 1192, 1879.

urements in confirmation of Stokes's law. In a still later paper Hagenbach[1] returned to the defence of Stokes's law as against Lommel[2] and Lubarsch,[3] who in the meantime had published further articles dealing with his objections and criticizing Lamansky's method. Wesendonck[4] in 1885 made observations with the sun's spectrum, using two concave mirrors and a prism, in the course of which he obtained very conclusive evidence that the fluorescence of naphthalin-roth extended to wave-lengths shorter than that of the exciting light. In 1886, Stenger[5] took the question up at length. He found that whether he used Hagenbach's method of illuminating the free surface, Lommel's method of grazing incidence through the side of a flask, or Lubarsch's fluorescent eye-piece, his measurements confirmed Lommel as to the invalidity of Stokes's law but not as to the independence of the fluorescent spectrum from the character of the exciting light. He also made experiments in colaboration with Hagenbach, who was finally converted to the same view.

It is our purpose in this paper to describe results obtained by the application of the spectrophotometer to the measurement of the fluorescence spectrum of those substances concerning the fluorescence of which, especially with reference to the validity of Stokes's law, the long-continued discussion already described arose. No attempt apparently has been made to apply the spectrophotometer to the study of fluorescence; yet it is obviously possible to determine both the limits and the maximum of a spectral region for which a curve of intensities can be plotted with far greater accuracy than by the method hitherto pursued by all observers, *i. e.*, that of attempting to set the cross-hair in the eye-piece of a spectroscope in the region of greatest brightness, or at the point where the spectrum ceases to be visible. Experimenters have perhaps been deterred from the use of the spectrophotometer because of the faintness of fluorescence spectra. It is true that fluorescent light from many substances is so weak as to preclude all measurements of its spectrum; but it is

[1] Hagenbach, Wied. Ann., 18, p. 45.

[2] Lommel, Wied. Ann., 8, p. 244, 1879.

[3] Lubarsch, Wied. Ann., 9, p. 665, 1890; also Wied. Ann., 11, p. 68, 1880.

[4] Wesendonck, Wied. Ann., 26, p. 521, 1885.

[5] Stenger, Wied. Ann., 28, p. 201.

also true, as we have found in the course of the experiments to be described, that settings can be made in cases where the brightness of the spectrum is far below that necessary to arouse the sense of color and where the presence of light can be detected only after prolonged shielding of the eye. The use of the cross-hair in such cases is out of the question, for the field is much too dim to render it visible, while every attempt to illuminate it from the side would flood the eye-piece with light sufficient to quench that under observation.

The instrument used in most of our observations was the spectrophotometer of Lummer and Brodhun. In order to secure the greatest possible sensitiveness to weak fields of view the ocular lenses in the eye-piece were used, the eye being focussed upon the aperture in the eye-piece and not upon the face of the prism. By means of metal screens attached to the collimator slits of the

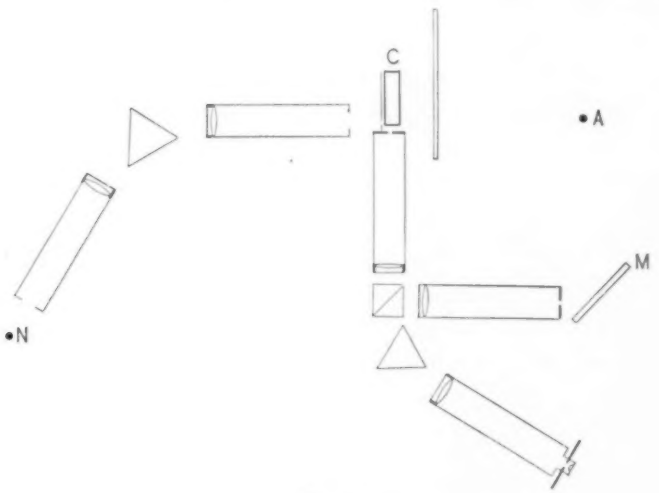

Fig. 1.

instrument the length of slit was regulated so as to avoid overlapping of the spectral images and to give two contiguous spectra in the field of view. One loses by this method the advantage of the method of contrast, but the sensitiveness of the instrument for low intensities is greatly increased. Since in most cases it was desired to employ monochromatic light for the excitation of fluorescence, the spectrophotometer was employed in connection with a large spectrometer as shown in Fig. 1. The eye-piece and slit of

the spectrometer were removed, and in place of the latter a Nernst filament (N) was mounted vertically. This filament afforded a nearly linear source of light, giving a sufficiently powerful continuous spectrum. The filament was attached to the arm carrying the collimator tube so as to move with the latter and to remain in the vertical plane passing through its axis. The observing telescope was clamped, and different portions of the spectrum were brought into the field as desired by movements of the collimator tube. The liquid, the fluorescence of which was to be studied, was placed in a rectangular cell (C) upon one face of which a real image of the spectrum was focussed. This face of the cell was provided with a metal screen having a vertical opening 1 mm. wide through which the light used for exciting fluorescence could pass. This opening (see Fig. 2) was so placed that the exciting light entering the cell

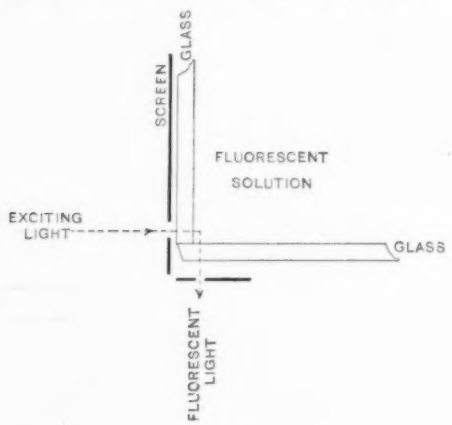

Fig. 2.

would be parallel to the adjacent face of the cell and as near to the same as practicable. In adjusting the metal screen one edge of the slit or opening was made to exactly cover the glass forming this face of the cell, as shown in the figure. The fluorescence spectrum was observed from a direction at right angles to the path of the exciting beam, and in order to bring the brightest fluorescence regions of the liquid into the field the vertical plane of the collimator of the spectrophotometer was adjusted so as to bring into the field of view the layer of liquid lying next to the face of the cell through which the exciting light entered. By this arrangement, the width of the

opening through which the exciting light entered the cell and the width of the slit of the spectrophotometer each being 1 mm., the average depth of liquid within which fluorescence was produced was approximately 0.55 mm. and the average distance which the fluorescent light passed through the liquid before leaving the cell was also 0.55 mm.

The source *A* of the comparison spectrum (Fig. 1) was an acetylene flame, the light from which was reflected diffusely from the face of the block of magnesium carbonate (*M*) mounted at an angle of 45° at the end of the collimator slit. Measurements were made by varying the width of the slit until the two regions of the spectrum under observation were equally bright.

The substances specified by Lommel as belonging to his first class, in which it is possible to excite fluorescence by means of light of wave-length longer than that of a portion of the fluorescence spectrum, and in which the distribution of intensities in the fluorescence spectrum is independent of the character of the exciting light, are *naphthalin-roth*, *eosin*, and *chlorophyll*. To this list Stenger added the substance *fluorescein*. The last-named substance, on account of its intense fluorescence and the location of the fluorescent band in the middle of the spectrum, in the regions of the highest luminosity, was selected for detailed study for the purpose of testing the conclusions reached by Lommel and the other investigators mentioned in the opening paragraphs of this paper.

Ten cubic centimeters of alcohol were saturated with fluorescein at room temperature and the solution was filtered. To 40 c.c. of distilled water one drop of a normal solution of sodium carbonate Na_2CO_3 was added. Two parts of the concentrated alcoholic solution were then mixed with 100 parts of the water thus rendered alkaline. The fluorescence spectrum of this solution, excited by the undispersed rays of the acetylene flame, was first measured; the distribution of intensities being compared, as in nearly all our subsequent experiments, with the spectrum of the light diffusely reflected from the surface of the the block of magnesium carbonate shown in Fig. 1. The absorption spectrum of the solution was then taken, the transmission through the cell, which had a thickness of 1.1 cm., being measured by means of the spectrophotometer.

The source of the transmitted light was a second similar block of magnesium carbonate illuminated by the same acetylene flame that served for the comparison spectrum.

Much stress having been laid by some of the previous observers upon the influence of stray light, the following measurements were made. The cell was filled with distilled water, set up precisely in the position in which it had been placed in the study of the fluorescence spectrum, and similarly illuminated by means of the acetylene flame. No measurable stray light was found, but an exceedingly weak fluorescence spectrum due to the glass walls of the cell was detected. Since the maximum of the fluorescence spectrum of the glass was found to lie further to the violet than the fluorescence spectrum of the fluorescein, and since moreover it was of scarcely measurable intensity, it was not deemed necessary to take further cognizance of these sources of error.

To determine the fluorescence spectrum of the solution when excited by monochromatic light, a mercury arc-light of the Lummer pattern was set up in front of the slit of the spectrometer. It was found that the violet lines from the spectrum of this arc produced fluorescence corresponding, as regards the position of the maximum and the general form, with the curve previously obtained by means of the light of the acetylene flame, but that the green line ($\lambda = .575$) excited no fluorescence. This latter result was to be expected since this light lies altogether outside of the absorption band of the solution in a region for which almost complete transparency exists. The spectrum of the acetylene flame was subsequently tried as an exciting source, but it was too weak to give easily measurable intensities of fluorescence. The slit of the spectrometer was then removed and a Nernst filament was mounted in the axis of the collimator tube as described in the specification of the apparatus and shown in Fig. 1. This filament was found to be of abundant brilliancy. It was maintained at constant brightness by means of a variable resistance, which was adjusted whenever the fluctuations of an ammeter placed in the electric circuit indicated it to be necessary.

Measurements of the fluorescence spectrum of the solution were made by means of the spectrum of the Nernst filament using as exciting light three nearly monochromatic regions of wave-length

$\lambda = .518\ \mu$ to $.536\ \mu$, $\lambda = .487\ \mu$ to $.502\ \mu$ and $\lambda = .460\ \mu$ to $.471\ \mu$. The curves thus obtained are plotted in Fig. 3, together with the

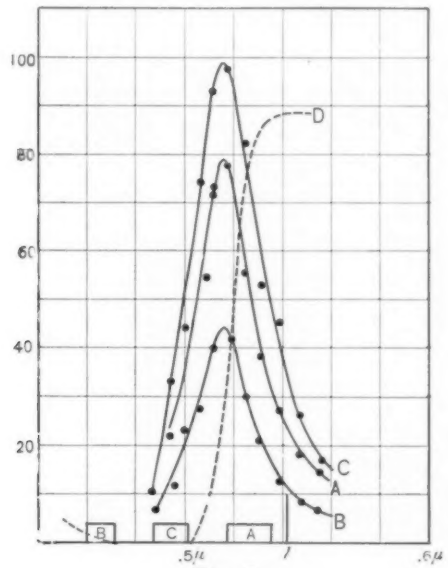

Fig. 3.

Fluoresceïn. Fluorescence spectra obtained when the exciting light lies in different regions of the spectrum. For curve *A*, the exciting light was confined to the region marked *A* on the horizontal axis, etc. Vertical scales arbitrary.

curve of transmission for the solution. It will be seen from this figure that the maximum of intensity of the fluorescence spectrum in these three cases lies in the same region at $.517\ \mu$ and that there is no evidence of any shifting of the fluorescence spectrum with the wave-length of the exciting light. It is obvious, moreover, that not only is it possible in the case of this solution to obtain fluorescence of refrangibility less than that of the exciting light, but that in the case of the curve marked *A* the maximum of the fluorescence spectrum is of shorter wave-length than the shortest wave-length used in excitation. These curves likewise agree fully in character and as regards the position of their maximum, with that for the fluorescence spectrum of the same solution when excited by the undispersed light of the acetylene flame. (Not shown in the figure.)

These curves for the fluorescence spectrum do not correspond precisely with the typical curve, meaning by that term the curve representing the distribution of intensities in the fluorescence spectrum of the surface layer of the fluorescing liquid. It is possible, however, in the case of a non-turbid medium, to compute from the observed curve the approximate form of the typical curve. Fig. 4 shows

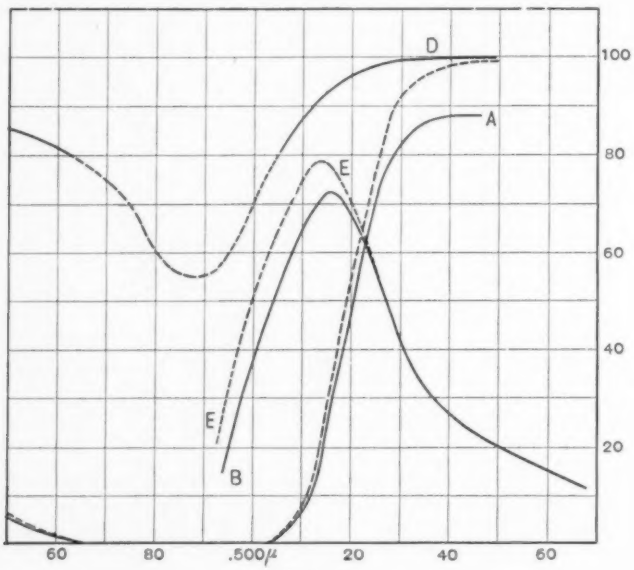

Fig. 4.

Fluorescein: typical fluorescence spectrum (*E E*).

graphically the result of such a computation.[1] Curve *A* gives the transmission of a glass cell containing a layer 1.1 cm. in thickness of the fluorescein solution. The dotted curve of similar form gives the transmission corrected for losses in the cell when filled with distilled water. From this by the well-known law of variation of absorption with the thickness, the curve *D* is found for a layer 0.055 cm. in thickness, which is the estimated mean distance through the solution which the fluorescent light passes before entering the slit of the spectrophotometer. The curve *B* is the observed curve of the distribution of the intensities in the fluorescence spectrum, and from this was computed the curve *E*, which represents, as nearly as the accuracy of the data will allow, the typical curve for this

[1] In this figure and in Figs. 5, 6, 7 and 8 the scale of wave-lengths has been doubled.

substance, corrected for absorption. It will be seen that in the case of this solution, under the conditions of the measurements, the absorption of the fluorescent light by the solution produces only a slight shifting of the maximum toward the shorter wave-lengths. When the fluorescent light passes through a considerable layer of the solution the effects of absorption are much more marked and there is a decided change of color. When a thin layer of the solution of fluoresceïn is viewed by reflected light its color is green, whereas the fluorescence of a mass of the liquid appears decidedly yellowish. This change is shown graphically in the curves of Fig. 5. In this figure *A* is the transmission curve

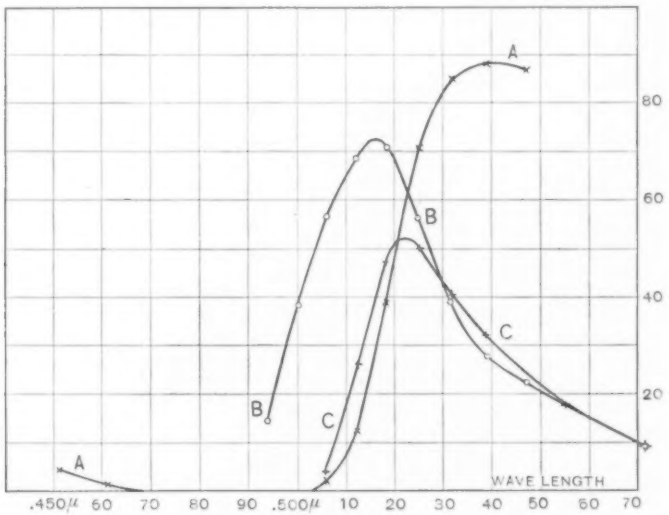

Fig. 5.

Fluoresceïn. Effect of absorption upon the fluorescence spectrum.

of the solution; *B* is the observed curve of fluorescence when the slit through which the excited light enters the cell is placed so that the fluorescent light passes through .055 cm. of the solution before exit; and *C* is the fluorescence curve when the slit is shifted to such a position that the fluorescent light passes through 1 cm. of the solution. It will be seen that the maximum is shifted from .516 μ to .522 μ, and that while the two curves are nearly coincident on the side towards the red the values on the other side of the curve fall off very rapidly as the result of the increased absorption. The

boundary of the fluorescence spectrum towards the violet in the one case would lie at about wave-length .505 μ, whereas in the thinner layer it would be visible to at least .490 μ. It is obvious that the color of the fluorescence in the latter case will contain a great excess of green.

The effect of diluting the fluorescent solution is similar to that of diminishing the distance through which the light passes. The results of observation upon a solution of fluoresceïn, diluted until the intensity of the fluorescence spectrum was diminished as far as would permit of satisfactory readings, are shown in Fig. 6. The

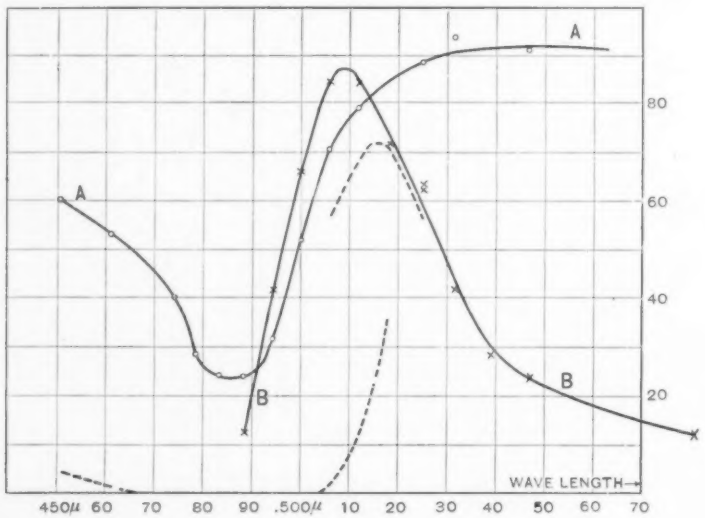

Fig. 6.

Fluoresceïn. Effect of dilution upon the fluorescence spectrum.

curve AA represents the transmission of the cell filled with the dilute solution and BB is the distribution curve of its fluorescence spectrum. The dotted lines show the corresponding transmission curve and a portion of the fluorescence curve for the solution before dilution. It will be noted that in this case, as in the case of the comparison of thick and thin layers of a given solution, the curves are coincident toward the red, but that the dilute solution has its maximum shifted towards the green ; also that the ordinates on this side of the curve show an increase indicative of the change of color, which, as is well known, is always observed as the result of diluting the fluorescent solutions of this substance.

Although, as has been shown in Figs. 4 and 5, the modifications produced by absorption in the curves of the fluorescence spectrum may be very marked, the effect of absorption diminishes rapidly with dilution of the solution. If we apply the correction for absorption, for example, to the curve for the dilute solution in Fig. 6 we find as is indicated in Fig. 7 that the change is insignificant. In

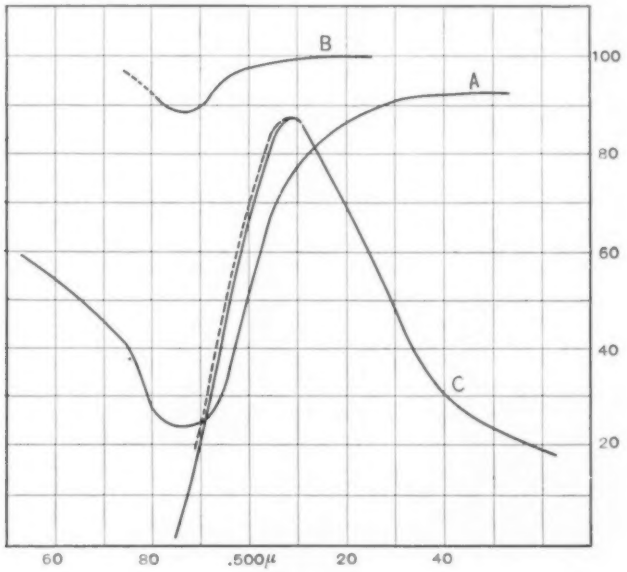

Fig. 7.

Fluorescein. Typical fluorescence spectrum for dilute solution.

this figure *C* is the observed curve for the fluorescence of the dilute solution, *A* the transmission curve of a layer 1.1 cm. in thickness, and *B* the computed curve for the transmission of the mean layer of liquid through which the fluorescent light has to pass. The correction for this absorption is indicated by means of the dotted line.

It having been established that the fluorescence of bodies of this class is independent, as regards the distribution of intensities, of the wave-length of the exciting source, it would be of interest to inquire whether the fluorescent energy for a given wave-length of the fluorescence spectrum varies with the wave-length of the exciting light, the energy of which is constant, or whether it depends only upon the energy. The rigorous determination of these relations

involves a knowledge of the distribution of energy in the spectrum of the exciting source, a difficult matter to determine with accuracy for the shorter wave-lengths of the visible spectrum. The curve shown in Fig. 8 may, however, be of some interest in this connec-

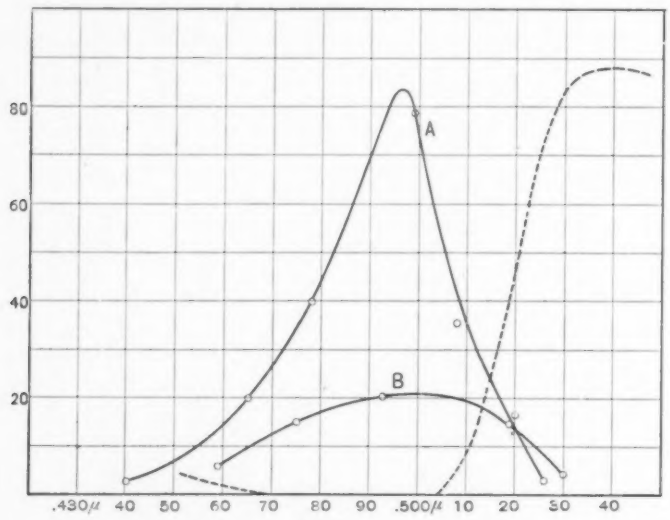

Fig. 8.

Fluoresceïn. Intensity of fluorescence as a function of wave length of exciting light. A refers to Nernst glower as source, B to acetylene flame.

tion. It represents the intensity of fluorescence, taken at the maximum of the fluorescence spectrum of the solution of fluoresceïn, as a function of the wave-length of the exciting light. Curve *A* was taken with the Nernst filament as a source, curve *B* with the acetylene flame. The dotted line shows the absorption band for a layer of the solution 1.1 cm. thick. It will be seen that fluorescence begins approximately at the wave-length at which the solution seems to be transparent, and that the maximum lies well within the absorption band but is shifted to the red. The longer wave-lengths within the band are more effective on account of their greater energy. The difference in the form of the curves *A* and *B* is probably ascribable to the different distributions of energy in the spectra of the sources of light employed.

In addition to the measurements on fluoresceïn the positions of the fluorescence spectra of solutions of eosin and naphthalin-roth

were determined by the method already described, and the transmission curves of the solutions were taken. Dilute solutions in alcohol were made, that of the naphthalin-roth being about $\frac{1}{200}$ saturated. The results obtained with these solutions, which are shown in Figs. 9 and 10, afford additional corroboration of the statements of Lommel. They are indeed in every respect analogous to those obtained with fluorescein and lead to the same conclusions. Each solution was excited by three distinct regions of the spectrum, one lying as far toward the red as practicable, one towards the blue and one at an intermediate wave-length. Curves showing the dis-

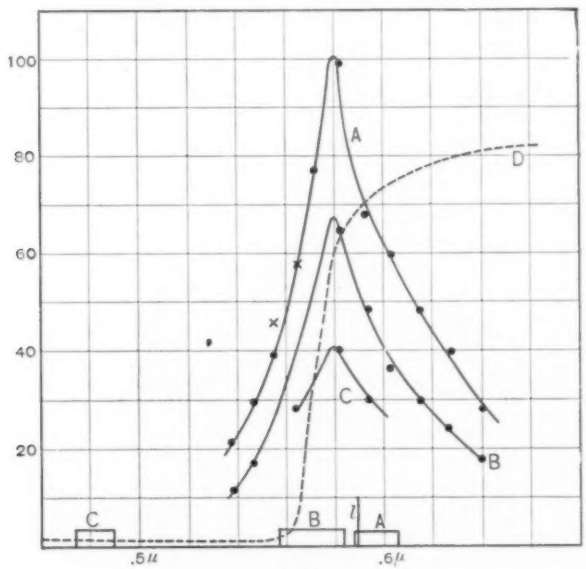

Fig. 9.

Eosin. Fluorescence spectra observed when the exciting light lies in different regions of the spectrum. Curve *A* was obtained when the exciting light was confined to the region marked *A* on the axis of wave-lengths, etc. Vertical scales arbitrary.

tribution of intensities in the three spectra thus produced are shown in the figures and it will be noted that, as in the corresponding curves for fluorescein, the position of the maximum is entirely independent of the wave-length of the exciting light and that the general character of the curve remains unchanged. In the case of these two solutions, as in that of fluorescein, it was possible to obtain a measurable amount of fluorescence by the use of light of

greater wave-length than that of the maximum of the fluorescence spectrum. The form of the fluorescence curve is very similar for these three substances but each has its own place in the spectrum. The maximum for fluorescein (Fig. 3) is at .517 μ, that for eosin at .580 μ and that for naphthalin-roth at .594 μ. The position of the maximum of these three curves with reference to the absorption band appears to vary with the different substances. The maximum for eosin coincides approximately with the infra-edge of the absorption band; that for fluorescein lies slightly (about .05 μ) towards the violet, while the maximum for naphthalin-roth is much further displaced towards the short wave-lengths.

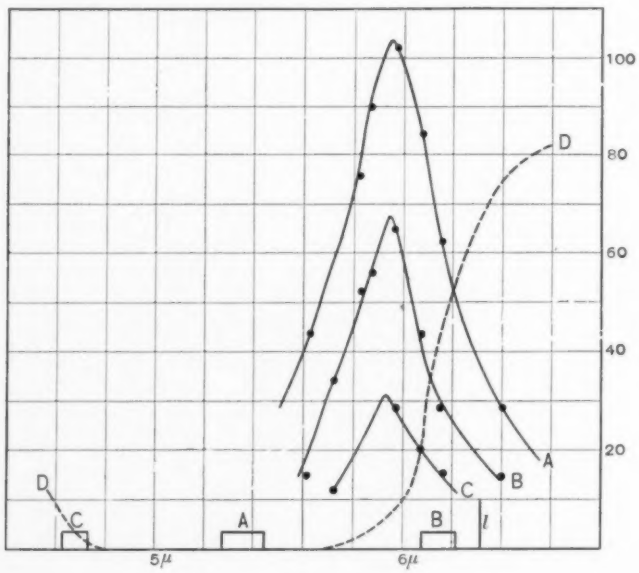

Fig. 10.

Naphthalin-roth. Fluorescence spectra obtained when the exciting light lies in different regions of the spectrum. Curve *A* was obtained when the exciting light was confined to the region marked *A*; etc. Vertical scales arbitrary.

Lommel's contention that it is possible to excite fluorescence in eosin by means of the light of the sodium flame is fully confirmed by the data plotted in Fig. 9, from which it will be seen that the exciting light by means of which curve *A* was obtained had a mean wave-length almost precisely equal to that of the sodium lines. Since it was found possible by means of light, all of which

was of greater wave-length than the maximum of the fluorescence spectrum, to produce fluorescence of sufficient strength for measurement with the spectrophotometer, it follows that *observable* fluorescence can be produced by light of even greater wave-length than that recorded in our diagram.

We deem the evidence already given in the foregoing paragraphs to be conclusive, so far as these substances are concerned; but in view of the differences of opinion among physicists as regards the validity of Stokes's law we venture to add the following description of a determination of the wave-length of the least refrangible monochromatic light which we found capable of exciting fluorescence in the three solutions with which this paper deals.

Since some writers have laid great stress upon the errors due to stray light, two different methods were employed of avoiding it. In the first the exciting light was passed through a solution of the substance to be examined before dispersion, thus filtering out those rays particularly active in producing fluorescence. The filtered light was then dispersed by means of the large spectroscope already described, and a second solution was subjected to an isolated, nearly monochromatic, region of the spectrum. The wave-length of this region was increased until the last trace of fluorescence, observed directly with the eye, was about to disappear. To distinguish between fluorescence and the presence of light diffused from small particles, the light was viewed at an angle of 90° through a Nicol prism, by which means diffuse light was completely excluded. The limit of excitation thus determined lay, as had been anticipated, further to the red than in the cases where a *spectrophotometrically measurable* fluorescence had been obtained, excepting in the case of eosin, where it was found to coincide almost exactly with the ultra-edge of the band used in exciting the spectrum shown in curve *A* (Fig. 9).

The second method consisted in sending the isolated region of the dispersed light to be used for excitation through a second spectroscope and determining as before the limit of excitability. This method of removing stray light has been extensively used and is well known to be effectual. The source of light in both cases was an arc-light.

The results obtained by these two methods were identical. They were as follows :

TABLE.

Substance.	Maximum Wave-length Exciting Observable Fluorescence.
Fluoresceïn,	.542
Eosin,	.589
Naphthalin-roth,	.632

These wave-lengths, all of which lie far to the red from the maximum of the fluorescence spectrum, are indicated in Figs. 3, 8, and 9 by means of the vertical lines marked *l*.

Measurements upon other fluorescent substances, and especially upon those which Lommel regarded as not belonging to his first class, are in progress and the results will be given in a subsequent communication.

PHYSICAL LABORATORY, CORNELL UNIVERSITY,
December, 1903.

THE VISCOSITY COEFFICIENT OF AIR, WITH AN INQUIRY INTO THE EFFECT OF THE RÖNTGEN RAYS THEREON.

By Frederick G. Reynolds.

I. Historical.

THE phenomena of viscosity have claimed the attention of investigators for many years ; but while the general truths may be said to be settled, there have always existed many differences of detail in which one may find a justification for further effort in the same field.

By the viscosity of a gas is meant the resistance which it offers to a continuous change of form divided by the rate at which that change is effected. The coefficient of viscosity may perhaps be best understood by considering a stratum of the gas infinite in extent. If this stratum were enclosed by two parallel planes at a distance d centimeters from each other and if one of these planes were to move parallel to the other with a velocity of v centimeters per second, keeping always in the same direction, till the gas embraced takes up its final velocity thus imparted, this velocity would decrease uniformly as we pass from the moving plane towards the other. If again the strata in contact with the two planes have the same velocity as the planes themselves, the strata will decrease in velocity v/d centimeters per second for every centimeter we depart from the moving plane. The friction between any two contiguous strata will then be the same as between either plane and the stratum in contact with it. If this friction be denoted by a tangential force F on every square centimeter of surface then $F = Uv/d$ where U is the coefficient of viscosity of the gas. The measurement of the internal friction or viscosity of a gas is the determination of the value of this coefficient.

The property which enables one layer to drag along another might in liquids be looked upon as a cohesion acting in opposition

to the motion, but in the case of a gas we must look for another explanation. In the light of the kinetic theory we look upon the gas as manifesting itself by the very rapid motion of its molecules. When two layers are moving therefore with unequal velocities, the molecules from the faster moving layer penetrate the slower moving one with their faster forward velocity and vice versa, so a measure of the friction would be the momentum which is thus carried over. The heat motion of the molecules is so great (being at 0° C., a mean velocity of 447 meters per second), that it is not affected by the comparatively slow forward motion of the molecules. It would naturally follow that this friction would increase with an increase of the temperature, for the speed of the molecules increases with the temperature being proportional to the square root of the absolute temperature. That the friction is independent of the density seems paradoxical at first, but the theory is at least justified when we consider that the transfer is made by the molecules and is greater as their number and activity are greater hence as the density is greater, but at the same time the transfer takes place in layers whose distance apart may be traversed by the molecules and as the density increases, this distance is restricted so that it is possible that the combined result of the two causes should produce a constant effect.

The methods of approaching and handling the investigations into the viscosity of fluids, both theoretical and practical may be divided into four general classes :

I. By their transpiration through capillary tubes. In this phase of the investigation Poiseuille [1] and Graham [2] were the pioneers, the former as to liquids and the latter as to gases.

II. By the swinging of pendulums. This method was followed by Baily.[3] Bessel [4] and Du Buat[5] whose work is discussed in a paper by Stokes[6] "On the internal Friction of Fluids on the Motion of Pendulums."

III. By the torsional vibrations of an immersed disc, first adopted by Coulomb.[7]

[1] Mém. de Savants Étrangers, 1846, IX., p. 433.
[2] Phil., Trans., 1846, CXXXVI., p. 573; 1849, CXXXIX., p. 349.
[3] Phil. Trans., 1832. [4] Berlin Acad., 1826
[5] Principes d'Hydraulique, 1786.
[6] Cambridge Phil. Trans., 1850, IX., pt. 2
[7] Mém. de l'Institut National, III., p. 246.

IV. By the torsional vibrations of a vessel filled with the fluid in question. This might be considered as a development of the preceding method.[1]

Each method seems to have its practical and theoretical advantages and disadvantages and the discordant results attending the early efforts in each have since been brought more closely into harmony through criticism and refinements of method. In speaking of the transpiration and torsion methods Meyer remarks of the discrepancies "als begründet in den Annahmen, welche zur Gewinnung brauchbarer Formeln in die theoretische Entwickelung eingeführt würden; die dabei eintretenden Vernächlässigungen wären in dem Falle schwingender Scheiben derart, dass der Coefficient zu gross, in dem Falle des Ausflusses durch Capillarröhren derart dass er zu klein erhalten müsste; die wahren Werthe würden also zwischen der Resultaten dieser beiden Methoden liegen." Which method comes nearer to the truth is not decided but the possible objection to the transpiration experiments lies in the difficulty of measurement of the diameters of the tubes and also in the fact that on account of the smallness of the bore one cannot be certain that the action between the molecules of the gas and of the substance of the tube does not affect the result. The pendulum method seems to be considered capable of great accuracy, but in the theory applicable to it Baily made the error of considering that the viscosity is dependent on the density. A revision of his results on the basis of a correct hypothesis has been promised but does not seem to have been made. The difficulties or objections connected with the method of a vibrating disc seem to be less than in the transpiration method and it may be for this reason that it has been the method generally selected by the later investigators; but the trouble here lies in the necessity of accounting properly for the effect of the motion of the medium near the edge of the discs. As a result of Maxwell's[2] analysis and deductions, given in 1860, both he and Meyer[3] started to practically demonstrate the truth of the same, using as a basis of their investigations the

[1] Helmholtz and Pietrowski, Sitzungsber. der kk. Akad., April, 1860.

[2] Phil. Mag., 1860, (4), XIX., p. 31; Phil. Trans., 1866, CLVI., p. 249.

[3] Pogg. Ann., 1865, CXXV., p. 177.

method adopted by Coulomb, but Meyer altered the earlier method in so far as to substitute three oscillating discs with a common axis instead of the one used by Coulomb. These plates were so arranged that it was possible to separate them, thereby exposing six surfaces, or to place them together, exposing only the two. In this way the friction on the exposed parts of the hanging system such as the mirror, etc., as well as the internal friction of the wire itself could be eliminated by considering the difference of the effects obtained when the plates were apart and together. Maxwell improved on this by inserting four fixed plates between the moving ones, which had the effect of causing the friction of the gas to exert a greater resistance and therefore one more easily measured, an advantage which was immediately recognized by Meyer[1] himself. A further advantage of Maxwell's method lies in the fact that his deductions give the viscosity coefficient directly; whereas Meyer's deductions lead to the square root which would cause the effect of any error to be increased. As a consequence Maxwell's results showed a better agreement in the effort to verify the independence of the coefficient and the pressure within the range of his observations, viz: from one to one sixtieth atmospheres. In 1887 as a result of a correction suggested by Koenig[2] an improvement was made by Meyer in his method and he calculated his results again on the improved theory.[3]

This independence of the viscosity and the pressure cannot be expected to hold in the limiting cases, as would follow from a consideration of the theory. The density and the free path enter we have seen as factors but when the former approaches o the latter approaches ∞ which would cause this phenomenon to fall outside of a development where only finite densities and free paths are considered.

Investigations on this point at low pressure were taken up by Kundt and Warburg,[4] who followed Maxwell's methods and found that below a pressure of one sixtieth atmosphere there was a

[1] Pogg. Ann., 1871, CXLIII., p. 14.

[2] Wied. Ann., 1887, XXXII., p. 193.

[3] Wied. Ann., XXXII., p. 642; Sitzungber. d. Münchener Akad., 1887, XVII., p. 343.

[4] Monatsber. d. Berl. Akad., 1875, p. 160; Pogg. Ann., 1875, CLV., pp. 337, 525.

marked diminution in the logarithmic decrement from which the viscosity coefficient is calculated. They however found an explanation of this in a phenomenon which they called slipping or (external friction) by which is meant the sliding of the medium along the bounding surface as distinguished from the sliding of one layer of the medium along another layer; and they explained this falling off as not caused by a decrease of the coefficient of viscosity but by an increase of the slipping at the lower pressure. Even a highly polished surface must in comparison with the molecules of the gas be considered rough, and under ordinary pressures a sliding along this, especially since the forward motion is very slow, is practically impossible, but this would not be the case in a very rare medium. By the term coefficient of slip they designated the ratio of the internal to the external friction, with the result that between the pressures of .6 and 20 millimeters of mercury the value of the coefficient of slip was found to be inversely proportional to the density of the gas and very nearly equal to the free path of the molecules. Therefore the external friction is directly proportional to the density. Warburg [1] experimented further on the external friction by the transpiration method and obtained smaller values for the coefficient of slip but Breitenbach [2] later obtained results nearer the earlier ones.

The correctness of this explanation of Kundt and Warburg for the sudden falling off the logarithmic decrement was later proven by Crookes [3] on a theory developed by Stokes,[4] wherein he used his vacuum tubes with vertically suspended strips of mica thereby excluding the possibility of any external friction. The results of these investigations show the independence of viscosity and pressure down to pressures so low as to be no longer accurately measured. However, at much higher vacuums there is a break in the constancy.

At extremely high pressures the law does not hold, but this might also be expected from the assumptions in the theory. Start-

[1] Pogg. Ann., 1876, CLIX., p. 399.
[2] Wied. Ann., 1899, LXVII., p. 826.
[3] Phil. Trans, 1881, CLXXII., p. 387.
[4] Phil. Trans., 1881, CLXXII, p. 435.

ing with the hypothesis that the particles traverse straight paths between their successive encounters with each other, the curved paths traversed during the period of actual encounter would be negligibly small, at ordinary densities or at low densities; but this would not be the case when the density becomes very great. The law considers the molecules unchangeable and the dissociation of the molecules under this extreme condition may contribute to this variation. This would be particularly true in the consideration of vapors. Warburg and Babo[1] have shown in the case of carbonic acid that the coefficient of viscosity increases with the density at pressures from 30 to 120 atmospheres, which law seems to hold for other gases as well.

In 1866 Meyer[2] published a theory where in assuming the independence of pressure upon the friction, he gave a law governing the speed of flow of gases through capillary tubes which corresponded with the law found by Poiseuille for liquids and which, by considering the amount of gas transpired in a given time leads to the value of the coefficient of friction. His calculations from the results of Graham's observations proved the correctness of his assumption. He obtained for the coefficient of viscosity of the air at 0° C. the values .000171, .000170, .000174 and he also gave the coefficients for nineteen other gases. Obermayer[3], Puluj[4] and Mutel following this method obtained for air the values .000167, .000180, .000172 respectively.

As to the fact of the increase of the viscosity with the increase of the temperature there can be no doubt, but as to the rate of such change there is not such an agreement among the different investigators. Maxwell first put it as increasing directly as the absolute temperature or proportional to $(1 + a\vartheta)$ where ϑ is the temperature from the freezing point and a is the coefficient of expansion. Later experiments show that for air at least it does not increase so rapidly. Efforts were made to represent the change by a factor of the form $(1 + a\vartheta)^n$ where n was first given as three fourths but later Barus[5]

[1] Weid. Ann., 1882, XVII., p. 390; Sitz. d. Berl. Akad., 1882, p. 509.

[2] Pogg. Ann., 1866, CXXVII., pp. 253, 353.

[3] Carl's Rep., 1876, XII., p. 13.

[4] Wien. Ber., 1874, LXIX., (2), p. 287; LXX. (2), p. 213.

[5] Bull. of the U. S. Geol. Surv., No. 54, Washington, 1889; Wied. Ann., 1889, XXXVI., p. 358.

adopting the transpiration method and with a range of temperature from 0° to 1300° gave to n the value two thirds for air and hydrogen. Different values were found by others for other gases and Wiedeman [1] found that n became smaller with the increase of temperature, a fact also found by Holman.[2] Schuman [3] however chose to represent the change by a formula consisting of two factors $(1 + \gamma\vartheta)^2\sqrt{1 + a\vartheta}$ from the consideration that the viscosity is dependent upon two things, the molecular free path and the molecular speed; and while this satisfied his own observations founded upon Maxwell's method, it did not agree with those of Barus. Sutherland [4] taking into consideration the *apparent* sphere of action of the molecules as distinguished from the real sphere of action, changed the first factor and represented the change by the equation,

$$\eta = \eta_0 \frac{(1 + ac)}{\left(1 + \frac{c}{\theta}\right)} \cdot \sqrt{1 + a\vartheta},$$

θ being the absolute temperature, a the coefficient of expansion and c a constant depending on the attractive forces exerted between the molecules when near together. With this he found results agreeing well with Barus and Holman.

Vapors as distinguished from gases proper have also been the subject of observations to determine if they follow the laws of gases as to viscosity. In these researches L. Meyer [5] and Koch [6] have followed the transpiration method while Puluj [7] and Schumann [8] have used the oscillation method with the result that at ordinary pressures, like true gases the coefficient of viscosity is independent of the pressure. However the formulæ given for true gases do not here represent exactly the change with the temperature and it seems that

[1] Arch. d. Sc. Phys. et Nat., 1876, LVI., p. 273.

[2] Proc. Amer. Acad., Boston, 1877, XII., p. 1; Phil. Mag. (5), III., p. 81; XXI., p. 199.

[3] Wied. Ann., 1884, XXIII., p. 353.

[4] Phil. Mag., 1893, (5), 202, p. 507.

[5] Wied. Ann., 1879, VII., p. 497; 1881, XIII., p. 1; 1882, XVI., pp. 369, 394.

[6] Wied. Ann., 1883, XIX., p. 857.

[7] Wien. Ber., 1878, LXXVIII., (2), p. 279; Carl's Report, 1878, XIV., p. 573.

[8] Wied. Ann., 1884, XXIII., p. 353.

the viscosity changes in a much larger ratio. Attempts to justify this disagreement have been made wherein the causes which make the laws of a perfect gas differ from the laws of vapors are used for a basis of argument. Of these perhaps the principal one which might be mentioned is the phenomenon of dissociation.

The question of the effect of electric and magnetic fields on the viscosity of a gas has not as yet been very satisfactorily determined, but that these cause no effect would appear from the results of Pagliani[1] and Noack.[2] Koenig[3] oscillated crystal spheres in a magnetic field but the effect on the coefficient of viscosity was not apparently considered. Quincke[4] also investigated the viscosity of fluids in the electric field, showing an increase without any apparent change in the period of swing of the spheres. To these results however objection was made by Boltzman.[5] The results of the efforts of investigators to determine the Coefficient of Viscosity of the Air, with the methods used by them may be seen by the table on p. 427.

As a consequence of the corrections in theory and through the refinements of process the results of the different methods while they varied widely at the start, have been brought into closer harmony. Since no new method differing in principle from these has been given, the only available way of testing the correctness of the results is in a variation of the methods at hand. The method of torsional vibrations seem to permit of a wider range of variation and therefore to be best suited to such a purpose. The objections to the use of discs however has been mentioned as lying in the allowance to be made for the action at the edge of the disc; but the improvement of method in substituting several in place of the one partly obviated this. This objection would not of course hold if a spherical surface were substituted and for this reason as well as that symmetry renders possible a rigorous theoretical development, the sphere or spherical shell seems best suited to answer the requirements.

[1] Ac. Torino, 1885, XX., p. 615; 1887, XXII., p. 1.
[2] Wied. Ann., 1886, XXVII., p. 289.
[3] Wied. Ann., 1887, XXXI., p. 273.
[4] Wied. Ann., 1897, LXII., p. 1.
[5] Wied. Ann., 1897, LX., p. 399.

Previous Determinations of the Coefficient of Viscosity of Air.

Investigator.	Method.	Coef. C. G. S.	Temp.	Reference.
Stokes	Baily's Pendulum	.000104		
Meyer	Oscillation	.000353	0	Crelle's Jour., 59, p. 229.
"	Bessel's Exper.	.000273		
"	Girault's "	.000384		
"	" "	.000360	18	
"	Oscillation (2°)	.000333	8.3	Pogg. Ann., 1871, CXLIII., p. 14.
"	"	.000323	21.5	
"	"	.000366	34.4	
"	Transpiration	.000168	0	Pogg. Ann., 1873, CXLVIII., pp. 37, 203.
"	"	.000174	0	
Maxwell	Oscillation	.000200	18	Phil. Trans., 1866, 156, p. 249.
Puluj	Transpiration	.000179	0	Wien. Sitz., Abth. 2, 1874, LXIX., p. 287.
				Wien. Sitz., Abth. 2, 1874, LXX., p. 243.
Von Obermayer	"	.000171	0	Carl's Rep., 1876, XII., p. 13.
"	"	.000168	0	Wien. Sitz., 1875, LXXI., p. 281; 1876, LXXIII., p. 433.
Schumann	Oscillation	.000168	0	Wied. Ann., 1884, XXIII., p. 353.
Schneebeli	Transpiration	.000171	0	Arch. des. Sci. Phys. et Nat. Geneve, 1885 (3), XIV., p. 197.
Tomlinson	Oscillation (Cylinder)	.000179	12.02	Phil. Trans. (2), 1886, CLXXVII., p. 768.
"	"	.000176	10.64	
"	"	.000177	14.63	
"	"	.000178	11.69	
"	" (Spheres)	.000176	9.97	

BIBLIOGRAPHY.

Baily.

Phil. Trans., 1832.

Barus.

Amer. Jour. Sci., 1888 (3), XXXV., p. 407.
Bull. of the U. S. Geol. Survey, No. 54, Washington, 1889.
Wied. Ann., 1889, XXXVI., p. 358.
Phil. Mag., 1890, No. 195, p. 337.

Bessel.

Berl. Akad., 1826, Math. Class., p. 1.

Boltzman.

Wien. Ber., 1872 (2), LXVI., p. 324; 1880 (2), LXXXI., p. 117; 1881 (2), LXXXIV., pp. 40, 1230; 1887 (2), XCV., p. 153; 1887 (2), XCVI., p. 891.
Wied. Ann., 1897, LX., p. 399.

Braun and Kurz.

Carls. Rep., 1882, XVIII., p. 569; 1883, XIX., p. 343.

Breitenbach.

Wied. Ann., 1899, LXVII., p. 803.
Annal. d. Phys, 1901, 310, p. 166.

Brodman.
Wied. Ann., 1892, XLV., p. 159.
Clausius.
Pogg. Ann., 1858, CV., p. 239.
Couette.
Compt. Rend., 1888, 107, p. 388.
Journ. de Phys., 1890, S. II., T. IX., p. 414.
Ann. de Chem. et. Phys., S. VI., T. 21, p. 433.
Coulomb.
Mem. de l'Institut, III., pp. 261, 287.
Crooks.
Phil. Trans., 1881, CLXXII., p. 387.
D'Arcy.
Mém. des divers savans, 1867, XV., p. 141.
De Heen.
Bull. de L'Acad. de Belgique, 1888 (3), XVI., p. 195.
Du Buat.
Principes d'Hydraulique, 1786.
Girault.
Mem. de l'Acad. de Caen., 1860.
Graham.
Phil. Trans., 1846, CXXXVI., pp. 573, 622; 1849, CXXXIX., p. 349.
Pogg. Ann., 1866, CXXVII., pp. 279, 365.
Grossman.
Inaug. Dis. Breslau, 1880.
Wied. Ann., 1882, p. 619.
Grotrian.
Pogg. Ann., 1876, CLVII., pp. 130, 237.
Wied. Ann., 1879, VIII., p. 529.
Guthrie.
Phil. Mag., 1878 (5), p. 433.
Hagen.
Pogg. Ann., 1839, XLVI., p. 423.
Abth. d. Berl. Akad., 1854, p. 17.
Hagenbach.
Pogg. Ann., 1860, CIX., pp. 385, 401.
Helmholtz (with **Pietrowski**).
Wien. Ber., 1860, XL., p. 607.
Heydweiller.
Sitz. d. Phys. Gesell. zu Berlin, 1895 (5), p. 33.
Hoffman.
Wied. Ann., 1884, XXI., p. 470.
Holman.
Proc. Amer. Acad. Boston, 1877, XII., p. 41; 1886, XXI., p. 1.
Phil. Mag., 1877 (5), III., p. 81; 1886, XXI., p. 199.
Houdaille.
Fortschr. der Phys., 1896, 52, Jahr. I., p. 442.
Klemenčič.
Carls. Rep., 1881, XVII., p. 144.
Wien. Ber. 1882 (2), 84, p. 146.
Beiblätertt, 1882, 6, p. 66.

Kirchoff.
Mechanik, 1877, 2. Aufl. 26, Vor., p. 381.

Koch.
Wied. Ann., 1883, XIX., p. 857.

Koenig.
Wied. Ann., 1885, XIV., p. 618; 1887, XXXI., p. 273; 1887, XXXII., p. 193.
Sitz. d. München. Ak., 1887, XVII., p. 343.

Kundt and Warburg.
Monatsber. d. Berl. Akad., 1875, p. 160.
Pogg. Ann., 1875, CLV., pp. 337, 525.

Lampe.
Programm des Städt Gym. zu Danzig, 1866.

Lampel.
Wien. Ber., 1886, XCIII. (2), p. 291.

Lang.
Wien. Ber., 1871, LXIV. (2), p. 485.
Pogg. Ann. 1871, CXLV., p. 290.

Margules.
Wien. Ber., 1881, LXXXIII., p. 588.

Maxwell.
Phil. Mag., 1860 (4), XIX., p. 31; 1868 (4), XXV., p. 209.
Phil. Trans., 1866, CLVI., p. 249.
Collected Papers, Vol. II.
Proc. Royal Soc., 1866, XV., p. 14.

Meyer, Lothar.
Wied. Ann., 1882, XVI., p. 394; 1885, XXV., p. 340.

Meyer, L., and Schumann.
Wied. Ann., 1881, XIII., p. 1.

Meyer, O.
Crelle's Journal, 1861, LIX., p. 237.
Pogg. Ann., 1861, CXIII., p. 85; 1865, CXXV., p. 177; 1866, CXXVII., pp. 253, 353, 367; 1871, CXLII., p. 513; 1871, CXLIII., p. 14; 1873, CXLVIII., p. 203.
Wied. Ann., 1879, VII., p. 497; 1881, XIII., p. 1; 1882, XVI., p. 394; 1887, XXXII., p. 642; 1891, XLIII., p. 1.
Phil. Mag. 1886 (21), p. 220.
Sitz. d. München Ak., 1887, XVII., p. 343.

Meyer, O., and Springmühl.
Pogg. Ann., 1873, CXLVIII., pp. 1, 503, 526.

Mützel.
Wied. Ann , 1891, XLIII., p. 15.

Navier.
Mem. de l'Acad. des Sciences, 1823, Tom. VI., p. 389; 1830, IX., p. 336.

Neuman.
Einl. in d. theor. Physik., p. 246.

Newton.
Phil. nat. princ. math., 1687, Lib. II., Sect. 9.

Noack.
Wied. Ann , 1886, XXVII., 289.

Obermayer.
Wien. Ber., 1875 (2), LXXI., p. 281; 1876 (2), LXXIII., p. 433.
Carls. Rep., 1876 (2), XII., pp. 13, 456; 1877, XIII., p. 130.
Pachu and Finazzi.
Nuovo Cimenti, April, 1900, XI., pp. 290, 294.
Atti del R. Instituto Veneto di Scienzi, 1900, LIX., pp. 389-403.
Pagliani.
Ac. Torino, 1885, XX., p. 615; 1887, XXII., p. 1.
Poiseuille.
Soc. Philomath, 1838, p. 77; Comptes rendus, 1840, XI., pp. 961, 1041; 1841, XII., p. 112; 1842, XV., p. 1167.
Ann. Chem., 1843 (3), VII., p. 50.
Mém. de savans étrangers, 1846, IX., p. 433.
Poisson.
Journal de l'école Polytech., 1831, XIII., p. 139.
Puluj.
Wien. Ber., 1874, LXIX. (2), p. 287; LXX. (2), p. 243; 1876, LXXIII. (2), p. 589; 1878, LXVIII. (2), p. 279.
Carl's Rep., 1878, XIV., p. 573.
Phil. Mag., 1878 (5), VI., p. 157.
Quincke.
Wied. Ann., 1880, X., pp. 161, 394, 513; 1883, XIX., pp. 545, 750; 1885, XXIV., pp. 347, 606; 1886, XXVIII., p. 529; 1888, XXXIV., p. 401; 1896, LIX., p. 417; 1897, LXII., p. 1.
Rells'ab.
Inaug. diss. Bonn., 1868.
Reynolds.
Royal Inst. of Gt. Brit., 1884, p. 1.
Beiblätter, 1886, X., p. 217.
Rutherford.
Phil. Mag., 1897, 209, p. 240.
Schmidt.
Wied. Ann., 1877, II., pp. 48, 241; 1882, XVI., p. 633.
Schultze.
Ann. d. Phys., 310, p. 140.
Schumann.
Wied. Ann., 1884, XXIII., p. 353.
Stefan.
Wien. Ber., 1862 (2), XLVI., p. 495; 1872 (2), LXV., p. 360.
Steudel.
Wied. Ann., 1882, XVI., p. 369.
Stokes.
Camb. Phil. Trans., 1849, VIII., p. 287; 1850 (2), IX., p. 8; 1881, CLXXII., p. 435.
Phil. Mag., 1851 (4), I., p. 337.
Sprung.
Pogg. Ann., 1876, CLIX., p. 1.
Sutherland.
Proc. Amer. Acad., 1885, pp. 13, 16.
Phil. Mag., 1893 (5), 202, p. 507.

Thomson and Rutherford.
Phil. Mag., 1896. 208, p. 392; 1893, 202, p. 313.
Tomlinson.
Phil. Trans., 1886 (2), 177, p. 767.
Warburg.
Pogg. Ann., 1876, CLIX., p. 403.
Warburg and Babo.
Wied. Ann., 1882, XVII., p. 390.
Sitz. d. Berl. Akad., 1882, p. 509.
Wiedemann.
Pogg. Ann., 1856, XCIX., p. 177.
Arch. d. Sc. Phys. et Nat., 1876, LVI., p. 273.
Fortschr. d. Phys., 1876, XXXII., p. 266.
Wilberforce.
Phil. Mag., 1891 (5), XXXI., p. 407.

(*To be continued.*)

THE EFFECT OF PRESSURE ON MAGNETIC INDUCTION.

BY FANNIE CORNELIA FRISBIE.

THE reciprocal relations existing between the change in shape or size of a body and its magnetization are so striking that they excite immediate interest, and make a thorough investigation of these relations necessary before a satisfactory theoretical explanation of the phenomena of magnetization can be given. As different investigators of the effect of hydrostatic pressure on magnetic permeability have obtained contradictory results, it was thought worth while to undertake the problem in a new way. Before describing the method used it will be well to survey briefly the theoretical and the experimental work already done.

On the supposition that Poisson's theory of induced magnetism for rigid bodies in a vacuum is correct, Helmholtz[1] (1881) deduced, with the aid of the principle of conservation of energy, expressions for the forces which act between the particles of magnetic or dialectric polarized bodies. These agreed with those of Clerk Maxwell[2] and Sir Wm. Thomson, except that they contained, beside the coefficient of induction, a constant which depends upon the nature of the medium, and which is determined by the change which it undergoes on change of density. Kirchoff[3] (1884) introduced another constant which makes it possible to express the change experienced by induction through dilations which are different in different directions, thus applying to solid bodies as well as to fluids.

Drude[4] has shown from purely theoretical considerations that since the shape of a body is altered by magnetization, any alteration

[1] Wied. Ann., 13, p. 400; Abhandlung, I., p. 798.

[2] Electricity and Magnetism, § 644.

[3] Wied. Ann., vol. 24, p. 52; Gesammelte Abhandlung, Nachtrag, p. 91.

[4] Physik des Aethers, p. 156.

of shape or size should change the value of the coefficient of magnetic permeability of the body.

It is found that change in the state of magnetism of a body alters its shape and size in various ways depending upon the strength of the magnetizing field.[1] The effect is to increase the length of an iron rod or ring in small magnetizing fields, as the field is made stronger the increase reaches a maximum, falls to zero, and for still stronger fields there is a diminution in length. It is found that accompanying the increase in length there is a decrease in thickness, and this led the early investigators to conclude that there was no volume change due to magnetization; later experimenters[2] have, however, found a rapid increase of volume with H, the magetizing force, to a certain point after which it increases asymptotically.

Since there was found to be a deformation of a body with an alteration of its magnetization, it was natural that one should next look for a reciprocal relation between the two phenomena, and it was found that a long thin rod under longitudinal tension underwent an increase of magnetization[3] until the magnetic moment reached a certain amount, after which further pull produced a diminution. Longitudinal and also transverse pressure produced effects opposite to those of longitudinal stretch.

The effects of hydrostatic pressure were investigated (1882) by Wassmuth[4] who placed the iron rod inside a thick walled tube which he filled with mercury. Pressure was obtained by the

[1] Wiedemann, Lehre von der Electricität, III., pp. 703–714. Joule, Phil. Mag., (3) 30, pp. 76, 225, 1847; reprint of papers, p. 235. Cantonne, Mem. d'R. Accad., vol. 6, p. 487, 1890. Bidwell, Proc. Roy. Soc., 40, pp. 109, 257, 1886; 43, p. 406, 1888; 47, pp. 46–49, 1890; 56, p. 94, 1894; Phil. Trans., vol. 179 A, 1888, p. 205; Beiblätter, 12, p. 600. Knott, Proc. Roy. Soc. Edin., 19, p. 249, 1892; 20, pp. 290, 295, 334, 1893–5; Trans. Roy. Soc. Edin., 38, p. 527, 1896.

[2] Nagaoka and Honda, Phil. Mag., 5, vol. 37, p. 131, 1894; 5, vol. 46, p. 261, 1898; Beibl. 18, p. 595; Wied. Ann., vol. 53, p. 487, 1894.

[3] Wiedemann, Lehre von der Electricität, III., pp. 698–703. Wertheim, Compt. Rend., 35, p. 702, 1852; Ann. de Chim. et Phys. (3), 50, p. 385, 1857. Matteucci, Ann. de Chim. et Phys. (3), 53, p. 416, 1858. Villari, Pogg. Ann., 126, p. 67, 1868. Sir Wm. Thomson, Phil. Trans., 166, p. 693, 1876; Beibl. 2, pp. 362, 607; Proc. Roy. Soc., 23, pp. 445, 473, 1875; 27, p. 439, 1878. Tomlinson, Proc. Roy. Soc. Lond., 42, p. 224, 1887; Beibl. 11, p. 659. Ewing, Beibl., 13, p. 186; Phil. Trans., 179, 333, 1888. Chree, Proc. Roy. Soc. Lond., 47, p. 41, 1899; Beibl., 14, p. 398; Phil. Trans., 181 A.

[4] Wien. Ber., 86, pp. 539, 550, 1882; Beibl. 7, p. 43, 1883.

expansion of the mercury when heated by means of warm water. One end of the glass tube was drawn out to a capillary from which increase of pressure was observed. The glass tube was surrounded by a solenoid which exerted a nearly uniform magnetizing force upon the iron rod. The intensity of magnetization was learned by the deflection of a mirror. He found that with increasing hydrostatic pressure the temporary moment in general decreased, and only increased for magnetizations which lie near the maximum. He estimated that he had about ten atmosphere pressure. Tomlinson[1] (1887) investigated the same subject and obtained negative results. He found no perceptible change either in the magnetic permeability of soft iron or in the permanent magnetism of hard steel; but at the beginning or at the end of the hydrostatic pressure, as similarly with longitudinal compression and extension, a part of the residual magnetism vanished. Nagaoka and Honda again obtained positive results, but these were not in agreement with those of Wassmuth. They had two similar brass tubes containing the rods to be examined. About the tubes were wound the magnetizing solenoids. The tubes were filled with water and pressure was obtained by means of a Cailletet pump. A magnetometer placed symmetrically with regard to the rods was adjusted until there was no deflection, then pressure was applied to one, and the change in magnetization produced a deflection of the magnetometer which could be observed. They used rods and ovoids of soft iron and found a decrease in the magnetization with pressure up to 250 atmospheres, using a magnetizing field, H, from 5–5.5 c.g.s. units. They calculated from data furnished by other parts of their series of experiments on magnetostriction the result to which Kirchoff's theory leads and found that the theory calls for an increase. Their series of experiments agreed with theory for the most part, but in regard to the effect of hydrostatic pressure there was not the same accord between experiment and theory. Unfortunately they make no statement about the condition of the iron and its previous treatment.

The present experiment may be divided into two parts: (1) The investigation of the effect of different pressures upon each of the rings with a constant magnetizing force; and also the influence

[1] Reference already given.

of residual magnetism upon the results. (2) The investigation of the effect of a certain pressure when magnetizing forces of different strengths are exerted upon the iron ; and also of the variation of results with change in the condition of the iron.

It was desired to avoid all end effects by the use of anchor rings, to secure higher pressure than the other experimenters had used and to test the effects with the same ring under various conditions and in different rings of the same material.

To obtain the pressure a special form of apparatus was designed (see Fig. 1) consisting of two parts of iron, 19 cm. outside diameter,

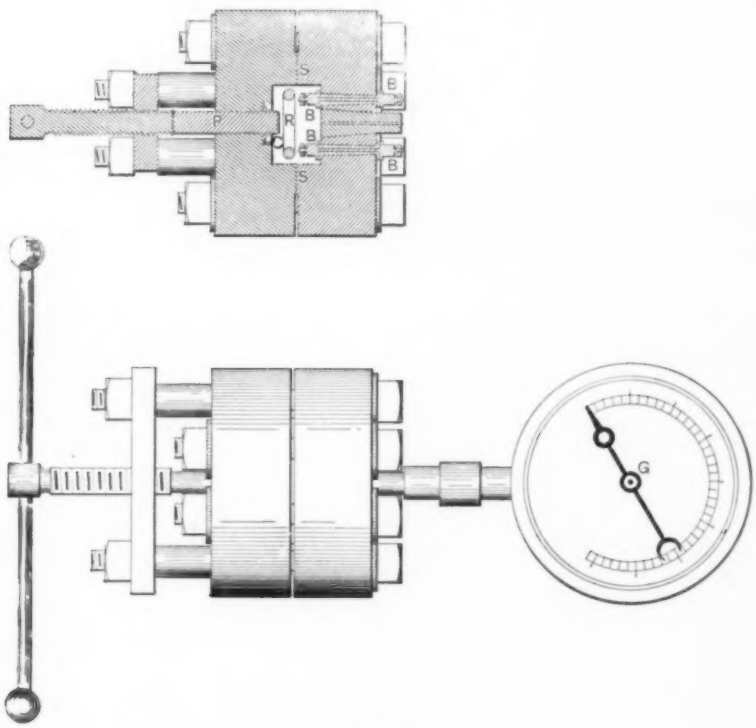

Fig. 1.

with walls 6.2 cm. thick. The contact surfaces (*S*) were well ground and the iron ring (*R*) was placed within, then the two parts were joined by means of eight five-eighth-inch bolts, thus making a thick-walled compartment. The connection of wires was made by means of four metal plugs terminating on either side in binding posts (*B*)

and insulated by taper plugs of hard rubber which passed through one end of the cylinder. A plunger in the other end was the means of obtaining pressure and the cavity was filled with oil through the hole for the plunger. Resin oil was used, which besides being very viscous is an excellent insulator. To avoid air bubbles the cylinder was first exhausted by an aspirator pump and the oil was then forced in by atmospheric pressure. A copper cap (C) prevented the leakage of oil around the plunger. Great difficulty was experienced in making the cylinders so tight that the desired pressure could be obtained without leakage. Various materials were tried between the halves but the tightest joint was finally secured by having the contact surfaces well ground. At first a slight smearing of vaseline and beeswax was used but later even this was found unnecessary. The pressure obtained was read directly from the gauge which registered as high as 18,000 lbs. per sq. in.

The three rings used were cut as nearly alike as possible from the same piece of Norway iron. They were wound with a secondary of No. 36 silk-covered copper wire, over which was a primary of No. 24 silk-covered copper wire. The layers were insulated from the iron and from one another by windings of insulating paper. Great care was exercised in winding that no wires should overlap. The dimensions and the data concerning the three rings used are given in the following table:

	Ring I.	Ring II.	Ring III.
Mean outer diameter.	5.46 cm.	5.41 cm.	4.45 cm.
Mean diameter of iron.	.624	.623	.661
No. turns in primary.	203	213	203
No. turns in secondary.	1,208	1,310	1,208

The galvanometer used was a D'Arsonval ballistic.

Part I. — Two rings were placed inside two pressure cylinders, so as to have both under the same conditions, and their primaries were connected in series with the desired number of cells of the storage battery, a resistance and an ammeter. The direction of throw of the galvanometer for each ring when the primary circuit was made or broken was noted; and the two secondaries were then connected in series with the galvanometer in such a way that the

two throws opposed each other. It was then observed which coil was predominant in the effect and a series of readings was taken giving the throw for both make and break of the primary. The plunger of the cylinder was then pressed in by means of the screw until the desired amount was registered by the gauge and another series of readings was taken for the throw under the given pressure, after which a new set of zero pressure observations was recorded. The difference in the deflection of the galvanometer was an index of the change of induction due to the pressure exerted upon the iron core.

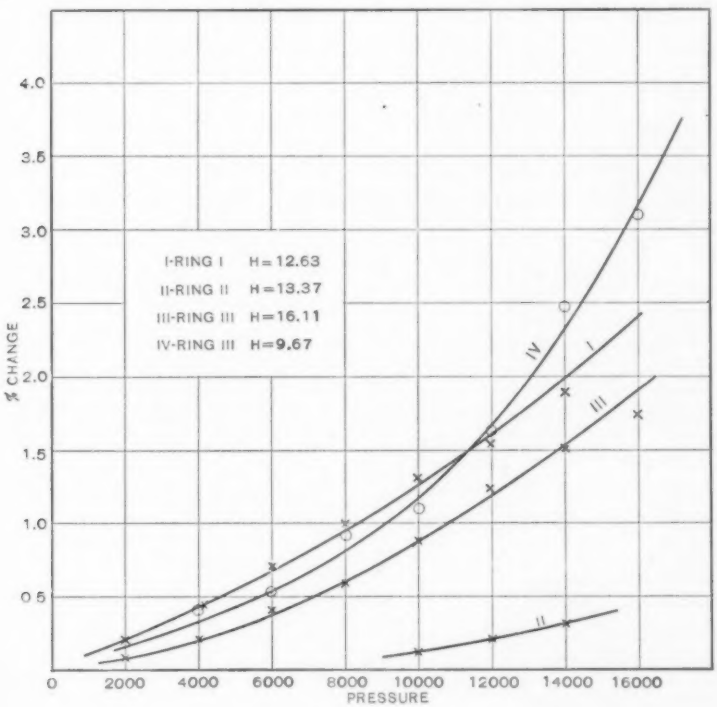

Fig. 2.

To ascertain the relative amount of this change the throw for each ring was observed when the same current was passed through the primary, and the secondary consisted of one or two turns; then the throw for the total number of turns, and the ratio of the change in induction to the total amount could be calculated.

In taking the readings it was found that the first throws after the

pressure had been put on or removed were not in agreement with the later ones. This accords with Tomlinson's observation that part of the residual magnetism is lost when pressure is applied or removed. Before recording the deflections, the circuit was made and broken a sufficient number of times to secure a steady magnetic condition.

A correction must be made for the volume change due to the pressure. To make this the value for the modulus of elasticity of wrought iron ($= 25 \times 10^6$) given by Wm. Kent, Handbuch, p. 314, was assumed and the correction made before the curves were plotted.

In Fig. 2, curves I., II., III., are shown the curves of the three rings, which represent the means of results of readings taken on different days. It is seen from the curves that although the percentage of change varies with the different pieces of iron, the effect of pressure is of the same general character for the three rings; and with a magnetizing force of from 12 to 16 absolute units, the magnetic permeability of iron in this condition is increased by hydrostatic pressure.

The following is a table of the results plotted in Fig. 2, curves I., II. and III. Centimeters change in deflection and the percentage this change is of the total deflection are given, also the corrected value, taking account of the variation of volume with pressure. The pressure is expressed in pounds per square inch. The magnetizing force, H, is in absolute units.

	I. $H = 12.63$.			II. $H = 13.37$.			III. $H = 16.11$.		
Pressure.	cm.	%	% cor.	cm.	%	% cor.	cm.	%	% cor.
2,000	.18	.20	.21				.08	.09	.10
4,000	.36	.40	.42				.18	.20	.22
6,000	.62	.68	.70				.35	.39	.41
8,000	.86	.95	.98				.51	.56	.59
10,000	1.15	1.27	1.31	.08	.08	.12	.76	.84	.88
12,000	1.35	1.49	1.54	.18	.17	.22	1.07	1.18	1.23
14,000	1.68	1.84	1.90	.26	.25	.31	1.32	1.46	1.52
16,000							1.52	1.68	1.74

To be sure that the change in the amount of the throw was not due to any alteration of temperature caused by the pressure, the

plunger was screwed in and the whole apparatus allowed to stand until any temperature difference would have been equalized by conduction, and the readings then taken agreed with the others.

The use of anchor rings made it possible to avoid end effects, but the method was open to the objection that the pressure was exerted upon the windings as well as upon the iron core itself, and it was necessary to be assured that the effect was not caused by this. A brass ring of the same dimensions as those of iron described above, and with similar windings on it, was subjected to the same treatment as the iron rings, but no change of deflection was observed. To test it in another way a balance of the throws was obtained by the insertion of an auxiliary solenoid, and then pressures of ten thousand and of fourteen thousand pounds were applied to the brass ring, and still no deflection of the galvanometer was observed. This showed that the change in deflection found before was not due to any changes produced in the windings or in the insulation by pressure.

Experiments proved that the position of the plunger within the ring had no effect upon the amount of throw, and that the number of layers of insulating paper between the successive layers of wire was immaterial, and also that the change in the compressed ring was independent of the opposing one and the second coil served only to cut down the deflection caused by the other so that the change could be observed.

It was known throughout the previous observations that the iron was probably almost saturated with magnetism, and for that reason one ring was carefully demagnetized by a gradually diminishing alternating current, and a new series of observations was made with a smaller value of H which would not give so near the maximum value of the intensity of magnetization. Great care was taken to keep the current steady as a slight change in the magnetizing force would cause a decided change in the magnetization. A change was also made in the method as the throw of the coil was diminished by resistance inserted in the secondary circuit instead of by means of a second coil. Each set of observations was taken with the current through the galvanometer first direct and then reversed. Curve IV., Fig. 2, shows the results obtained for ring III. after it had

been demagnetized and treated in this manner, which shows that when the iron is not saturated with magnetism the percentage change is greater.

Results plotted in curve IV., Fig. 2, are given in the following table, where pressure is in pounds per square inch and the percentage change observed as well as the value corrected for volume change is recorded. Ring III. was used after demagnetization and $H = 9.67$.

Pressure.	Per cent. Change.	Per cent. Corrected.
4,000	.39	.41
6,000	.51	.53
8,000	.89	.92
10,000	1.06	1.10
12,000	1.58	1.63
14,000	2.42	2.48
16,000	3.05	3.11

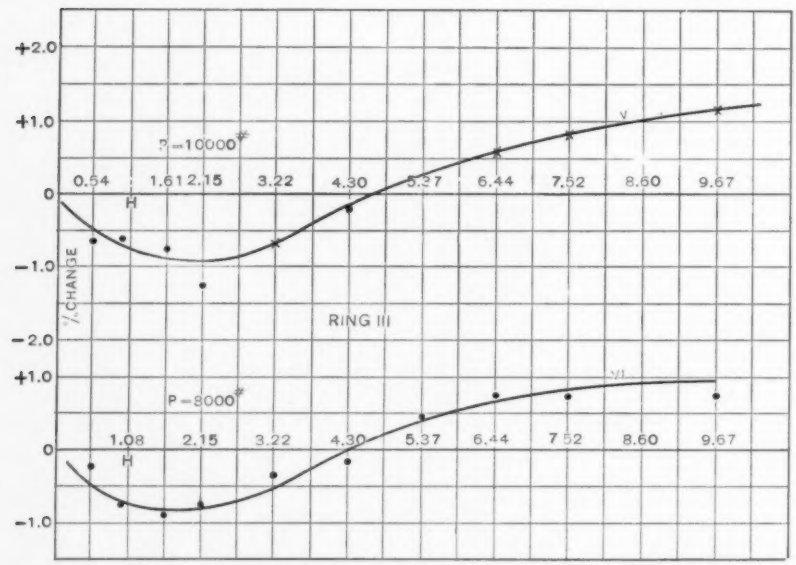

Fig. 3.

Part II. — As in the last observations of Part I. only one ring was used, and the deflection was cut down by the insertion of resistance in the secondary circuit. Instead of using the same current and taking the changes in deflection for various pressures, the current was varied for each set of readings and the pressure was

each time made the same. The results are shown in Fig. 3, curves V. and VI. V. shows the percentage change of magnetic induction produced in ring III. by pressure of ten thousand pounds

Table for Fig. 3. Curves V. and VI.; Current in Amperes and Values of H in Absolute Units.

Amperes.	*H*	Curve V. 10,000 Pounds.		Curve VI. 8,000 Pounds.	
		% Change.	% Corrected.	% Change.	% Corrected.
.025	.54	− .67	− .63	−.24	−.21
.045	.97	− .66	− .62	−.76	−.73
.075	1.61	− .77	− .73	−.90	−.87
.100	2.15	−1.31	−1.27	−.78	−.75
.150	3.22	− .71	− .67	−.36	−.33
.200	4.30	− .19	− .15	−.18	−.15
.250	5.37	+ .05	+ .09	+.41	+.44
.300	6.44	+ .56	+ .60	+.73	+.76
.350	7.52	+ .78	+ .82	+.69	+.72
.450	9.67	+1.11	+1.15	+.70	+.70

per square inch, when the magnetizing force varied from .54 to 9.67 c.g.s. units. VI. represents the effect of eight thousand pounds pressure for the same ring after demagnetization.

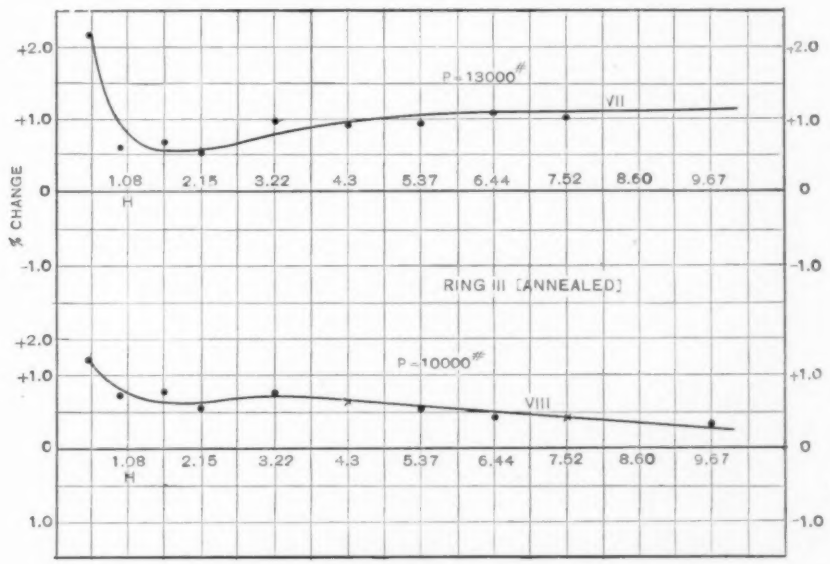

Fig. 4.

It is seen in both cases that with a small magnetizing force the magnetic induction decreases, reaching a minimum for a field

whose strength lies between 1.6 and 2.1 absolute units. This diminution grows less with increasing fields, until for a value of H about 4.5 or 4.8 it shows a zero change. Further strengthening of the magnetizing field results in an increase, which seems to approach a maximum for H equal to approximately 9 c.g.s. units with eight thousand pounds pressure, and for a slightly larger value when the pressure is ten thousand pounds.

After the ring had been very carefully annealed another series of observations gave the results represented in curves VII. and VIII., Fig. 4. For VII., as the iron had just been annealed, there was no need of demagnetization; the pressure applied was thirteen thousand pounds per square inch. Before the readings for VIII. were taken the ring was demagnetized; the pressure in this case was ten thousand pounds. The table of results plotted is the following:

Amperes.	H	Curve VII. 13,000 Pounds.		Curve VIII. 10,000 Pounds.	
		% Change.	% Corrected.	% Change.	% Corrected.
.025	.54	+2.10	+2.15	+1.14	+1.18
.045	.97	.54	.59	.68	.72
.075	1.61	.64	.69	.74	.78
.100	2.15	.46	.51	.55	.59
.150	3.22	.94	.99	.70	.74
.200	4.30	.89	.94	.63	.67
.250	5.37	.90	.95	.51	.55
.300	6.44	1.05	1.10	.37	.41
.350	7.52	.97	1.02	.37	.41
.450	9.67			.32	.36

It is interesting to note on comparison of these values with those obtained for the same ring before it was annealed, that in this case the change is an increase for every value of H used, and that, with the exception of the first value which was obtained with a very small current, the percentages of change are included within narrower limits for the annealed iron. It is seen that the second, third and fourth points of the curve seem to give a slight wave, but it was felt that the results were hardly sufficiently accurate to warrant drawing it in. Curiously the effect of ten thousand pounds pressure diminishes for greater strengths of the field.

Throughout the observations great care was taken to keep the conditions steady. The current was watched carefully, and was

maintained at the desired value by a slight alteration of the resistance in the primary circuit. The pressure was kept as constant as possible during the time required for the observations.

The experiment seems to warrant the following conclusions:

1. The application of hydrostatic pressure to soft iron anchor rings changes the coefficient of magnetic permeability of the iron, besides altering the amount of residual magnetism when the pressure is applied or removed. The amount of the change varies for different pieces of iron.

2. With unannealed iron the sign of the change depends upon the strength of the magnetizing field; the effect in small fields being a decrease, and in stronger fields an increase; the change of sign occurs for $H = 4.5$ or 5 c.g.s. units. The magnitude of the alteration is a function of both field and pressure.

3. With annealed iron an increase is found for pressure with all values of field between .54 and 9.67 absolute units.

The results with the unnealed iron are in agreement with those obtained by Wassmuth in his experiment, as there is a decrease and then an increase for values of H approaching the maximum of magnetization. But for the values of the field used by Nagaoka and Honda there is clearly an increase for iron, annealed and unannealed whereas they obtained only a decrease in magnetization, which was not the result called for by Kirchoff's theory and their own experiments on other forms of magnetostriction. The results of this investigation would then tend to support the theory of magnetism advanced by Kirchoff for annealed soft iron.

The author wishes to express her indebtedness to Professor Michelson, of the University of Chicago, for suggesting the problem, and the method of attack, and for his aid throughout the investigation; to Professors Mann and Millikan for their encouragement and assistance during the work; and to Dr. Jewett who assisted in taking the final observations, and made the drawings for this article.

BARNARD COLLEGE.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE TWENTY-FOURTH MEETING.

THE spring meeting of the Physical Society was held at the Cosmos Club, Washington, D. C., on Friday, April 22, and Saturday, April 23, 1904.

The following papers were presented:

1. A Study of the Silver Voltameter. K. E. GUTHE.

2. Some new Rectifying Effects in Conducting Gases. P. G. NUTTING.

3. The Effect of Light on the Absorption and Electrical Conductivity of Fluorescent Liquids. E. L. NICHOLS and ERNEST MERRITT.

4. Some Additions to the Arc Spectra of the Alkali Metals. F. A. SAUNDERS.

5. The Volumes of Solutions. W. F. MAGIE.

6. Absolute Electrodynamometers. G. W. PATTERSON.

7. Induced Radioactivity due to Radium. E. P. ADAMS.

8. The Energy Density, the Tension, and the Pressure in a Magnetic Field. (Read by title.) S. J. BARNETT.

9. A Recomparison of the U. S. Prototype Meter at the International Bureau of Weights and Measures. L. A. FISCHER.

10. (*a*) High Temperature Measurement by means of Optical Pyrometers. (*b*) Note on Special Problems in Optical Pyrometry. C. W. WAIDNER and G. K. BURGESS.

11. Apparatus for Platinum Resistance Thermometry. C. W. WAIDNER and H. C. DICKINSON.

12. Intercomparison of Primary Standard Mercurial Thermometers. C. W. WAIDNER and H. C. DICKINSON.

13. The Standard Cell. F. A. WOLFF.

14. The Peculiar Behavior of Some Resistance Standards and Its Explanation. F. A. WOLFF.

15. A Direct Reading Apparatus for the Calibration of Resistance Boxes. F. A. WOLFF.

16. Absolute Measurement of Capacity. E. B. Rosa and F. W. Grover.

17. Absolute Measurement of Inductance. E. B. Rosa and F. W. Grover.

18. The Testing of Mica Condensers. E. B. Rosa and F. W. Grover.

19. Testing of Alternating-Current Instruments. (Read by title.) E. B. Rosa and M. G. Lloyd.

20. Equilibrium of Vapor Pressure over Curved Surfaces. (Read by title.) A. A. Bacon.

Adjourned at 1 P. M., April 23.

E. B. Rosa,
Secretary pro tempore.

A Study of the Silver Voltameter.[1]

By K. E. Guthe.

THE unsatisfactory results obtained with the common type of silver voltameter have led to the impression that this instrument cannot be regarded as very reliable. Recently Richards with his students and Leduc have attempted to improve the instrument, and claim to have obtained a high degree of accuracy. The present investigation was undertaken in order to compare different forms and subject them to a severe test.

Richards has pointed out that the main trouble arises from the formation of a complex ion at the anode which, when transported to the kathode, will deposit too much silver. To prevent the anode liquid from reaching the kathode, he surrounds the anode, consisting of a silver rod, by a porous, fine-grained cup and keeps the level of the electrolyte in this cup always lower than that of the outside. Leduc, on the other hand, hopes to prevent the formation of the disturbing ion by using a very small current density at the anode, and he therefore employs as anode a mass of granulated silver surrounded by filter paper.

The experiments described show (1) that the usual type of silver voltameter is not a very reliable instrument for measuring quantity of electricity; (2) that Leduc's type does not prevent the formation of the complex ion, even with small anode current density; (3) that the use of filter paper must be avoided in the silver voltameter; (4) that silver in contact with the anode liquid breaks up the complex ion.

The Richards type is the only satisfactory form so far proposed and gives very constant results. For convenience in manipulation this type

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 22, 1904.

has been modified in such a way as to avoid the necessity of removing, by frequent pipetting, the anode liquid from the bottom of the porous cups. Finely divided silver was placed at the bottom of the cup and this effectually removed the complex ion. In order to insure a constant resistance of the voltameter it was found necessary to surround the lower part of the porous cup by a small glass beaker.

A second type proposed by the author is the large silver anode voltameter. Granulated silver is placed in a layer about 0.5 cm. deep at the bottom of a large porous cup; upon this is pressed the silver plate forming the anode connection. The whole is immersed into the platinum bowl (the kathode) containing neutral 20 per cent. solution of silver nitrate.

To test the degree of accuracy which may be obtained with these voltameters in a number of independent experiments, the instruments were arranged in series with a standard resistance, and the current adjusted so as to give a terminal potential difference equal to the E.M.F. of a Weston cell, which was used throughout the investigation. In the different experiments resistance standards of 5, 2, and 1 ohm were used, and in one case a 2 and a 1 ohm placed in parallel, so that currents ranging from 0.2 to 1.5 amperes were employed. In all experiments a little more than 4 grams of silver was deposited at the kathode.

The results obtained with the Richards type and the two designed by the author were very consistent, the largest variation of any single value from the mean not exceeding 1 in 20,000, except when a current of only 0.2 amperes was used. The experiments in this case lasted five hours and the deposits became too heavy due on account of diffusion of the anode liquid through the cups. In the experiments lasting a shorter time no such difficulty was experienced.

The electrochemical equivalent of silver, expressed in terms of the present standards of the Bureau—assuming that the Clark cell has an E.M.F. of 1.434 volts at 15°C., according to the legal specifications—was found to be by—

The Richards type	1.11682 mg. per coulomb.
The modified Richards type	1.11682 " " "
The large Anode type	1.11683 " " "

Relative comparison with other forms gave for the usual type 0.048 per cent., for the Ag_2O type, employed by Patterson and Guthe 0.058 per cent. larger deposits.

This would increase the electrochemical equivalent of silver as determined by means of the former to 1.11736, and for the Ag_2O voltameter to 1.11747 mg. per coulomb; this agrees well with the results of Kahle, who calculated, taking the electrochemical equivalent as 1.118 mg., the E.M.F. of the Clark cell to 1.43305 volt. On the basis stated above the electrochemical equivalent would be reduced to 1.11726 mg.

It seems highly desirable that in future experiments, especially in absolute measurements of current, the type now in common use be abandoned. Any one of the three forms mentioned above will allow a degree of accuracy which cannot be obtained by the other method.

BUREAU OF STANDARDS,
WASHINGTON, D. C.

THE EFFECT OF LIGHT UPON THE ABSORPTION AND THE ELECTRICAL CONDUCTIVITY OF FLUORESCENT SOLUTIONS.[1]

BY E. L. NICHOLS AND ERNEST MERRITT.

IT is a general characteristic of fluorescent bodies that the light emitted during fluorescence is different in color from the light which the body absorbs, and to which the fluorescence is due. Although the absorption spectrum and the fluorescence spectrum overlap they do not coincide. In fact the fluorescence spectrum often extends to wave-lengths for which the absorption of the substance, as ordinarily measured, is practically *nil*. Since we are accustomed to the view that absorption and emission are converse phenomena, and that under like conditions a substance absorbs the same rays that it is capable of emitting, it is therefore natural to expect that the absorbing power of a fluorescent body will be modified by fluorescence.

Such an effect has, in fact, been observed by Burke,[2] who found that the absorption of uranium glass was about 30 per cent. greater when the glass was brightly fluorescent than when its absorption was determined under ordinary conditions. The light used in Burke's experiments was that due to the fluorescence of a second piece of uranium glass.

The writers have recently investigated the influence of fluorescence upon absorption in the case of a solution of fluoresceïn. Measurements were made by means of a Lummer-Brodhun spectro-photometer for a number of different wave-lengths throughout the region in which the effect was observable. Upon plotting the results it was found that the effect of fluorescence is to introduce a new and well-defined band into the absorption spectrum of the solution, the extent of this band being approximately that of the spectrum of the fluorescent light. In the case of a rather dilute solution the maximum increase in absorption occurred at wave-length .507 μ. When fluorescence was excited by a group of four acetylene burners the increase in absorption at this point amounted to 67 per cent. In a more concentrated solution the region

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 22, 1904.

[2] Philosophical Transactions, Vol. 191 (A), p. 87, 1898.

of greatest increase in absorption lies further toward the red, and coincides closely with the edge of the normal absorption band. The percentage absorption due to fluorescence was found to be a function of the intensity of the incident light, being greatest when the intensity of the incident light was least.

The action of light upon a fluorescent substance thus produces a distinct change in its behavior. It appears to us not unlikely that this change is to be ascribed to some type of temporary dissociation brought about by the exciting light. If this view is correct fluorescence might result from the violent oscillations that would probably occur, not only at the instant of dissociation, but also during the subsequent recombination. A temporary change in the absorbing power of the substance would also be expected. From the standpoint of the electromagnetic theory of light a type of dissociation which results in splitting up the molecule into positive and negative ions seems not only possible but probable, especially in the case of substances possessing well-marked absorption bands. Such a dissociation might be of the ordinary electrolytic type, or it might be similar to that which occurs in gases under the influence of Roentgen rays. In either case it would produce an increase in electrical conductivity.

We have investigated the effect of fluorescence upon conducting power in the case of five fluorescent solutions, each of which shows a small but perfectly definite increase in conductivity when illuminated. During the test the solutions were contained in a glass cell whose parallel vertical sides were about 0.1 mm. apart. The electrodes were pieces of platinum foil about 20 mm. long. They were mounted vertically and were approximately 2 mm. apart. In order to make the normal conducting power small, absolute alcohol was used as a solvent. The solutions were made so concentrated that the fluorescence was confined to a thin layer at the surface. Under these conditions the normal resistance of the cell, without illumination, ranged from 1,000,000 ohms in the case of fluorescein to 90,000 ohms in the case of fuchsin. The electric arc was used as a source of light. Direct illumination of the cell by the undispersed rays of the arc could not be employed on account of the heating effect; even when the rays were passed through a water bath the change in resistance due to rise in temperature was considerably greater than that due to fluorescence. The solution to be tested was therefore placed in the spectrum of the arc, and no further disturbance due to temperature effects was experienced. No change in resistance was observed except when the rays falling on the solution were capable of producing fluorescence; and the maximum increase in conductivity occurred in the region where the fluorescence was brightest. For the different substances tested the observed increase in conductivity had the following values: Eosin, 1.1 per cent.; fluorescein,

0.11 per cent.; rhodamin, 0.14 per cent ; naphthalin roth, 0.06 per cent.; cyanin, 0.59 per cent. In the case of an alcoholic solution of fuchsin, which was not fluorescent, no change in resistance was observed, although the conditions were such that a change of .008 per cent. could have been detected.

The Volumes of Solutions.[1]

By William Francis Magie.

WHEN a solution is made by dissolving a salt or some other electrolyte in water, the volume of the solution is always less than the sum of the volumes of the water and of the solid which was dissolved. There are cases in which the volume of the solution is less than that of the water which enters into it. In such cases certainly a condensation of some or all of the water must occur. We may conceive of the volume of the solution as made up of three parts ; (1) the volume of that portion of the water which is not within the sphere of influence of the solute, or which is not associated with it ; (2) the volume of the undissociated molecules of the solute and of the water associated with them ; (3) the volume of the dissociated ions of the solute and of the water associated with them. From this conception it follows that the volume of the solution may be represented by the formula

$$V = W + A(n - p) + Cp,$$

in which W represents the volume of the water used ; n, the number of gram-molecules of solute ; p, the number of these gram-molecules which are dissociated (in case $n = 1$, p represents the dissociation factor) ; and A and C are quantities which depend respectively on the undissociated molecules of the solute, and on the dissociated ions. The formula is most conveniently tested by setting $n = 1$, $C - A = B$, and using $V = W + A + Bp$. Comparison with the experimental data shows that for each solute A and B are constants over a considerable range of dilution.

A formula of this form has been applied by MacGregor[1] to the density of solutions. The density, however, is not an additive property, and the constants obtained do not admit of interpretation. On the other hand, we may fairly assume that, so long as the solution is so dilute that a considerable portion of the water is outside the range of action of the solute, the volume of the solution is strictly an additive property.

Others of MacGregor's results are obtained by the use of the formula on additive properties, and confirm the conception of a solution held in this paper.

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 22, 1904.

[2] Phil. Mag., Jan.-June, 1897.

From the data given by MacGregor for the density of sodium chloride solutions we may calculate the amount of water present in the solutions of various strengths, and also the constants A and B. The results obtained when the formula is used to calculate the volumes of the different solutions are exhibited in the following table. In this table the column headed n contains the number of gram-molecules of sodium chloride in one liter of solution. The column headed V contains the calculated volumes of the solutions, which, by experiment, should be uniformly 1,000 c.c.

Sodium Chloride.

$A = 26.06$, $B = -12$.		$A = 26.06$, $B = -12$.	
n	V	n	V
0.25	999.89	1.5	999.55
0.50	1,000.04	2.0	1,000.08
1.00	1,000.00	2.5	999.99

The volume of one gram-molecule of solid sodium chloride ranges, according to the various determinations of the density, between 26 and 27. The value of A is therefore equal to the molecular volume of the solid salt, and we may conclude that the volume of the undissociated molecule of salt is not changed on solution, and that, if the molecule of salt is associated with molecules of water their volume is not changed thereby.

Calculations on data furnished by Kohlrausch and Hallwachs[1] confirm the validity of the formula in a similar manner. In most cases the calculated results obtained depart from the observations when the value of n is greater than 1.

Thomsen[2] furnished data expressed in a different way, by which also the validity of the formula may be illustrated. In the following tables the column headed N contains the number of gram-molecules of water used to dissolve one gram-molecule of the solute. The column headed p contains the dissociation factor, obtained from Kohlrausch's tables of conductivity. The other columns contain the observed volumes of these solutions and the volumes calculated with the formula.

Sodium Hydroxide.

$A = 7$, $B = -15$.			
N	p	Obs.	Calc.
30	0.602	537.9	538.0
50	.696	896.4	897.0
100	.775	1,795.9	1,795.4
200	.824	3,594.8	3,594.7

[1] Wied. Ann., 50, 1893.

[2] Thermochemische Untersuchungen, vol. 1.

The molecular volume of solid sodium hydroxide is 19, greater than the value of A. There appears to be a considerable shrinkage either of the undissociated molecule or of water molecules associated with it.

Ammonium Chloride.

$A = 43.5$, $B = -7.5$.

N	ϕ	Obs.	Calc.
10	0.593	219.0	219.1
25	.703	488.2	488.2
50	.742	937.8	937.9
100	.777	1,837.7	1,837.7
200	.806	3,637.6	3,637.5

The molecular volume of the solid ammonium chloride is 35, less than the value of A. The molecule appears to increase in volume on solution or to affect the associated water molecules so as to increase their volume.

Similar calculations on other data given by Thomsen confirm the validity of the formula in a similar manner.

The constant $C = A + B$ of the formula depends on the dissociated ions of the solute and on the water associated with them. In case the solute is one which divides into two ions, this constant is the sum of two similar constants, C_c and C_1, which depend respectively on the two ions. If, now, we know the values of C for two solutes which have a common ion, and if the constant C_0 depending on this common ion has the same value in both solutions then the difference of the values of C for the two solutions, or $C_1 - C^2$, will depend only on the other two ions, and will be the same for any pair of compounds of those two ions with a common ion. This conclusion from the hypothesis is illustrated by the results contained in the following table, in which are given the values of C for three pairs of compounds of potassium and sodium with common ions. The differences of the values of C for each pair are very nearly equal.

Values of C and Differences.

KCl	23	KOH	1	KNO_3	35.5
NaCl	14	NaOH	−8	$NaNO_3$	24.4
Difference,	9		9		11.0

It has been shown [1] that a formula of the same form as the one illustrated here applies to the heat capacities of solutions. Since the heat capacity is also manifestly an additive property of a composite body, this serves to confirm the conception of the nature of a solution from which the formula was deduced. The results in general also con-

[1] Bull. Am. Phys. Soc., vol. 2, no. 2.

firm the correctness of the dissociation factors obtained by the measurement of conductivity.

It is very desirable to obtain a set of tables of conductivities, or of the corresponding dissociation factors, for solutions made up by adding one gram-molecule of the solute to a known number of gram-molecules of the solvent. The theoretical formulas which apply to solutions imply the use of such solutions, and the illustration of those formulas by the data which exist at the present time involves tedious and somewhat uncertain calculations. Such tables would be far more convenient for the use of the physicist than those in which the strength of the solutions is given in gram-molecules per liter.

THE ENERGY DENSITY, TENSION AND PRESSURE IN A MAGNETIC FIELD.[1]

BY S. J. BARNETT.

THE general expression for the magnetic energy density at a point at which the induction is proportional to the intensity is obtained by equating the volume integral of the energy density in the field of a simple circuit in a medium in which the induction is everywhere proportional to the intensity to half the product of the current in the circuit (expressed as $\frac{1}{4}\pi$ times the line integral of the magnetic intensity around it) and the magnetic flux through the circuit, and comparing the two expressions.

The expressions for the magnetic tension and pressure at a point at which the induction and intensity are proportional and parallel are simply and rigorously obtained by the application to simple circuits of Maxwell's relations between the change in the electrokinetic energy of a system, with constant fluxes or constant currents, and the work done by the electromagnetic forces when the system undergoes an infinitesimal change of configuration. The magnetic pressure is also deduced from Ampère's law.

SOME ADDITIONS TO THE ARC SPECTRA OF THE ALKALI METALS.[1]

BY F. A. SAUNDERS.

LENARD[2] discovered by means of a slitless spectroscope a new series and several other lines in the spectrum of sodium. The object of the present investigation was to see if these could not be found

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 22, 1904.

[2] Ann. d. Ph., 11, p. 636, 1903.

by ordinary methods if the apparatus gave spectra free from astigmatism and with sufficient economy of light. A Rowland concave grating was used, of 10 cm. width and 305 cm. radius, ruled with unusually long lines on a parabolic surface. The light from the slit was rendered parallel before reaching the grating by falling on a 10 cm. parabolic mirror. The spectra were formed 150 cm. from the grating, along a curve which is nearly a circle of 75 cm. radius; they were free from astigmatism, at least in part, and were, when photographed on a film of suitable curvature, of very good definition over a space of 25 cm., corresponding to about 2,700 tenth-meters in the first order.

The purchase of this apparatus was made possible by the generosity of the Rumford Fund Committee.

Photographs were obtained showing Lenard's new series in the sodium spectrum and one or two new lines not mentioned by Lenard. The series lines could be seen as pairs of diffuse lines, the stronger line having the greater wave-length. Careful measurements were taken of the wave-lengths of these lines. Two new terms were added to the series; one in the deep red (taken by a three-hour exposure on a film stained with cyanin) and another in the blue. The series pairs have constant wave-number differences, the separation being about 0.85 that of the subordinate series. No formula has as yet been found to fit the series unless it has four arbitrary constants; those of Kayser and Runge, Rydberg, Fowler and Ritz were tried, along with others, modifications of these.

Two new lines have been found in the spectrum of lithium, which when combined with the line at 4,636 measured by Hagenbach[1] form a new series, also apparently of the subordinate type. Its lines accompany those of the first subordinate series, perhaps as "satellites" though their separation from the parent lines is considerable. They are diffuse and broadened toward the red. The lithium lines near 4,602 appear to admit of a different interpretation from that given by Hagenbach. It seems easiest to explain them as consisting of one line at 4,603.2, broadened toward the red, which is a member of the first subordinate series, and a line at 4,601.6, broadened toward the violet, analogous to the pair in the sodium spectrum at 5,676 and 5,670.

In the spectrum of potassium one new line has been found, predicted by Ritz,[2] in almost exactly the predicted place (6,964.7). It is one of the hitherto missing first pair of the first subordinate series, which is, for some obscure reason, very faint. The other member of this pair was not found, owing to the broadening of the line at 6,938.8 near which it doubtless lies.

[1] Ann. d. Ph., 9, p. 48, 1902.
[2] Ann. d. Ph., 12, p. 264, 1903.

Two new lines were found in the spectrum of rubidium; one a principal series line at 3,158.7 (not seen as a pair) and a very diffuse and faint line at 5,171, which with 5,234.6[1] forms a pair in the second subordinate series. No other new lines were found though the spectrum was well photographed down to 8,000 *t-m*.

In the cæsium spectrum five new lines were found, one at 7,944.7, a member of the second subordinate series, predicted by Ritz, and the others members of pairs of lines with constant wave-number differences, which, combined with lines previously observed and not belonging to the regular series, form two new series of the subordinate type, running to the same end. This end is not, however, the end of the regular series also, but is somewhat nearer the red. These lines are so exceedingly diffuse that their positions cannot be measured with much accuracy. It seems most unlikely from their appearance that they can be due to any impurity. It does not appear to have been previously noticed that the heavier line of each pair in the first subordinate series is accompanied by a companion (lines all observed by Ramage) on the red side, forming a "secondary" series.

In all the above spectra several lines in the deepest red were carefully measured whenever it seemed that the existing measurements could be improved upon. The results differ considerably from those of Lehmann,[2] in some cases by several tenth-meters. Special precautions were taken to avoid errorrs and check measurements and it is thought that in this case the "probable" error and the "possible" error are neither more than a few tenths of a tenth-meter.

THE STANDARD CELL.[3]

BY F. A. WOLFF, JR.

IN the paper is briefly discussed the main proposition, which will be taken up by the Chamber of Delegates in the St. Louis International Electrical Congress — the redefinition of the fundamental international electrical units.

At the Chicago Congress the ohm was defined in terms of the resistance of a column of mercury of specified length and mass and at a specified temperature; the ampere as a one tenth of a C.G.S. electro-magnetic unit and as being represented sufficiently well by a current which deposits silver at a specified rate under specified conditions; the volt in terms of

[1] Observed by Ramage, Astrophys. Jour., 16, p. 43, 1902.

[2] Ann. d. Ph., 5, p. 638, 1901.

[3] Abstract of a paper presented at the Washington meeting of the Physical Society, April 22, 1904.

the ohm and ampere and as being defined sufficiently well in terms of the Clark standard cell, assuming for the electromotive force of the latter at 15° Centigrade the value of 1.434 volts.

While it was supposed that these definitions would serve as the basis for legislative enactment by the various countries represented, few have taken definite action and nearly all have adopted definitions not in strict accord with the above. This may be attributed to the following reasons:

All three of the fundamental units are defined in terms of concrete standards, notwithstanding that they are connected by Ohm's law.

The specifications for the silver voltameter were shown to be entirely inadequate.

The assumed value for the electromotive force of the Clark cell was shown to be too high by about 1 in 1,400.

The application of potentiometer methods and the improvement in the methods for measurement of low resistance standards have set a standard of accuracy for relative measurements of current and electromotive force far in excess of that fixed by the reproducibility of the fundamental standards.

The St. Louis Congress will therefore most likely be called upon to define two of the three fundamental electrical units in terms of concrete standards, in which case the third will be defined in terms of the other two. The definition of the ohm is almost free from criticism, and the choice for the second unit lies between the ampere defined in terms of the electrochemical equivalent of silver and the volt as defined in terms of the electromotive force of some standard cell. The great superiority of the Weston cadmium cell over the Clark cell would seem to point to the entire displacement of the latter. The arguments for and against both propositions are reviewed, the deciding argument in favor of the adoption of the standard cell being the necessity of using the same as a working standard at least in actual practice, owing to the limited range of application of the silver voltameter, even if the reproducibility of the cell should be somewhat less than the accuracy of silver voltameter determinations. As the reproducibility of the cell depends upon the purity of the materials employed and principally upon the mercurous sulphate, an electrolytic method for the preparation of the latter material was devised, consisting in the electrolysis of a five per cent. sulphuric acid solution between mercury electrodes, the solution and the mercury anode being agitated by means of a glass stirrer. (Practically the same method was described independently by Carhart and Hulett and by the author at the annual meeting of the Electrochemical Society on April 7, 1904.)

Experiments w th Weston cells set up from this material and with single electrode cells showed that the electrolytically prepared material gave a constant electromotive force to within one or two parts in 100,000

and values about two tenths millivolts less than the best commercial samples of chemically pure mercurous sulphate washed with water in accordance with the Chicago specifications. This difference was shown to be due to the basic mercurous sulphate, either in the samples or produced by the action of the water, as the same samples when treated with ten per cent. sulphuric acid and washed with concentrated neutralized cadmium sulphate solution gave values almost in agreement with those obtained from the electrolytic material. In addition, samples of electrolytic mercurous sulphate treated with water, thus rendering it basic, gave values about two tenths millivolts higher than when the sample was washed directly with concentrated neutralized cadmium sulphate solution. The cells prepared with the new material seemed to come to a state of equilibrium immediately after setting up, which, as is well known, is not the case with the cells prepared from commercial materials, all of which contain at least traces of basic mercuric sulphate. This may be attributed to the presence of this material, which is slowly reduced in the cell and which is also removed by washing with ten per cent. sulphuric acid, and which if added to the electrolytic mercurous sulphate gives values considerably in excess of the normal values.

A DIRECT READING APPARATUS FOR CALIBRATING RESISTANCE BOXES.[1]

BY F. A. WOLFF, JR.

IN precision comparisons of the coils of resistance boxes, the variable plug and other contact resistances introduce serious errors unless special methods involving a determination of such connecting resistances or their elimination are employed.

The method described by the author consists of determining the relative values of the coils belonging to a decade or to adjacent decades by shunting these coils in series with each other by a series of 100-ohm coils, the galvanometer being connected in turn across corresponding points of the two series. The apparatus is made direct reading by applying a Kelvin-Varley shunt to four adjacent coils of the auxiliary series, the shunt having a resistance of 400 ohms and including in its middle portion a bridge wire having a range of $\pm$ 1 per cent.

The relative values of the coils are given by the differences in the readings obtained, and it is therefore only necessary to determine the value of one of the coils of the series, to determine the values of

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 22, 1904.

all in international ohms. The method is applicable to boxes in which the coils may be put in series and in which a galvanometer contact may be applied to the contact blocks to which the coils are attached. In the series plug rheostats this may be accomphished by means of a split plug, one half of which is composed of insulating material.

The Peculiar Behavior of Some Resistance Standards and Its Explanation.

By F. A. Wolff, Jr.

SOME years ago it was noticed by Professor T. C. Mendenhall and by Professor Benjamin F. Thomas, that certain paraffin insulated resistance coils showed polarization effects due to the application of the test current. The same behavior is noticed occasionally, even with coils insulated by shellacing and subsequent baking, and is to be attributed to an electrolytic shunt, the polarization of which causes an apparent increase in the resistance as the test current is left on. On the reversal of the test current the resistance is apparently too low, but gradually increases to its former value. The effect is produced by dipping a coil which does not show the effect in water and is eliminated by subsequent baking.

Another effect, which has not been described before, manifests itself by a decrease in resistance as the test current is left on, the amount of decrease depending upon the voltage applied to the coil. The variation is irreversible, depending upon the previous currents and their direction, and the behavior is exactly similar to that of a coherer resistance. The effect cannot be accounted for by thermal or voltaic electromotive forces, and probably consists of a coherer shunt to a portion of the coil, due to a metal filing imbedded in the shellac between turns. As Branly has shown, such coherer effects may be obtained even with solid dielectrics.

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 22, 1904.

NEW BOOKS.

The Induction Motor, Its Theory and Design Set Forth by a Practical Method of Calculation. By HENRI BOY DE LA TOUR. Translated from the French by C. O. Mailloux. McGraw Publishing Co., New York, 1903. Pp. 1 + 200.

This admirable book comprises a most thorough analysis of the induction motor. The graphical and mathematical treatment is concrete and is made practical by application to numerical cases. It is well adopted to the uses of designing engineers and engineering students.

The translator, while respecting the individuality and originality of the author, has shown considerable interest in making the English presentation of the subject as good as possible. He is to be commended for his careful use of terms, for the introduction of "torque" (contrary to the general custom of continental writers) and for the use of the "maxwell" and the "gauss" as units of magnetic flux and density. We look askance at the statement: "The terms *watted power* and *wattless power* are self explanatory." Both author and translator show themselves to be strong believers in the induction motor. Both are to be congratulated for this comprehensive presentation of the subject.

F. BEDELL.

Conductors for Electrical Distribution; their Materials and Manufacture, The Calculation of Circuits, Pole-Line Construction, Underground Working, and other Uses. By F. A. C. PERRINE. D. Van Nostrand Company, New York, 1903. Pp. 1 + 280.

In this volume the subject of electrical conductors is treated from two aspects, that of manufacture and that of engineering, two aspects so distinct that we would criticise the bringing of the two together were it not for the fact that the author is peculiarly fitted to treat of each.

About half the book deals with the art of wire making. Here is a fund of information which the author's experience as manufacturer makes authoritative.

The remaining half deals with engineering and here too the stamp of the writer is shown. Well known work, as Kelvin's law of economy of conductors is clearly treated. The chapters on line calculations form the most valuable part of the book. The method of treatment first given

by Perrine and Baum before the American Institute of Electrical Engineers in 1900 is given in full. This treatment, which is practically correct, is much simpler than that previously given by various writers on the theory of lines with distributed self-induction and capacity.

The book is commended to the student and the transmission-line engineer.

F. BEDELL.

Engineering Preliminaries for an Interurban Electric Railway. By ERNEST GONZENBACH. McGraw Publishing Co., New York, 1903. Pp. 1 + 71.

This is a brief report on a proposed railway in the Middle West. The author states that it is not to be construed as an attempt to generalize, but it is intended to show the way in which certain conditions were to be met in a certain case. The field of the book is obviously limited.

F. B.

Notes on Electric Railway Economics and Preliminary Engineering. By W. C. GOTSHALL. McGraw Publishing Co., New York, 1903. Pp. 1 + 248.

The author expresses his intention to consider his subject broadly and as a whole, to outline the work of the electric railway engineer during the inaugurative and constructive period of a proposed road, and to give in a general way some bases upon which the costs of construction, probable traffic and ultimate economic results of the proposed railway can be guaged. The analysis of the subject is well made and the treatment clear.

F. B.

INDEX TO VOLUME XVIII.

A.

Aërial systems, resonance in, *Taylor*, 230.
Air, Viscosity Coefficient of, Reynolds, 419.
Alcohol, methyl, electrical conductivity of solutions in, *Kraus*, 40, 89.
Alexander, *Applied Mechanics*, 375.
Alkali Metals, Arc Spectra of, Saunders, 452.
American Physical Society, Minutes of, 116, 295, 444.
Andrews, E. J., *Elements of Physics*, 379.
Arc, electric, Hall effect in, *Child*, 370.

B.

Barnes, H. T., Heating effect of radium emanation, 118.
Barnett, S. J., The Energy Density, Tension and Pressure in a Magnetic Field, 452.
Polarization and recovery of cadmium cells, 104.
Bedell, F., *de La Tour's Induction Motor*, 458.
Perrine's Conductors for Electrical Distribution, 458.
Distribution, 458.
Gonzenbach: Interurban Electric Railways, 459.
Gotschall: Electric Railway Economics, 459.
Book reviews:
Astrophysical problems, *Clerke*, 378.
Batteries, secondary, *Wade*, 384.
Conductors for Electrical Distribution, *Perrine*, 458.
Electric Railway Economics, *Gotschall*, 459.
Entropy, *Swinburne*, 380.
Electrical machinery, *Ryan, Norris and Hoxie*, 382.
Induction Motors, *de La Tour*, 458.
Inte urban Electric Railways, *Gonzenbach*, 459.
Light for students, *Edser*, 192.
Mechanics, *Alexander and Thomson*, 375
Mineralogy, *Miers*, 63.
Physics, *Andrews and Howland*, 379.
Physics, laboratory manual, *Cheston, Dean and Timmerman*, 64.
Thermodynamik, Voigt, 372.
Brace, D. B., Half shade elliptical polarizer and compensator, 70.
Briggs, L. J., Conductivity of liquid films, 123.
Bumstead, H. A., *Swinburne's Entropy*, 380.
Burton, E. F., Radio-activity of ordinary metals, 188.

C.

Cadmium cells, polarization recovery of, *Barnett*, 104.
Calorimetry, steam, continuous method of, *Hart*, 263.
Carbon resistances, convenient method of mounting, *Gray*, 57.
Cell, The Standard, *Wolff*, 454.
Cells, cadmium, polarization recovery of, *Barnett*, 104.
Cheston, H. C., *Laboratory Manual of Physics*, 64.
Child, C. D., Hall effect in electric arc, 370.
Clerke, A. M., *Astrophysical Problems*, 378.
Condenser field, alternating, radiation produced in, *Sanford*, 366.
Conduction in mercury vapor, *Wills*, 319.
Conductivity, electrical, of solutions in methyl alcohol, *Kraus*, 40, 89.
Conductivity of liquid films, *Briggs and McLane*, 123.

Conductivity produced in rarefied gases by an incandescent kathode, *Merritt and Stewart*, 239.
Compensator, and polarizer, half shade elliptical, *Brace*, 70.
Cook, S. R., Theory of electrolytic rectifier, 23.
Cornish, R. H., *Andrews and Howland's Elements of Physics*, 379.
Cheston, Dean, and Timmerman's Laboratory Manual of Physics, 64.
Crehore, W. W., *Alexander and Thompson's Applied Mechanics*, 375.
Critical point, electrical conductivity of solutions in methyl alcohol in the neighborhood of, *Kraus*, 40, 89.

D.

Dean, P. R., *Laboratory Manual of Physics*, 64.
Differential telephone, *Duane and Lory*, 275.
Doubt, T. E., Effect of intensity upon velocity of light, 129.
Duane, William, Differential telephone, 275.

E.

Edmunds, C. K., Metallic reflection of selenium, 193.
Reflecting Power of Selenium as determined by a Spectrophotometer, 385.
Edser, Edwin, *Light for Students*, 192.
Electric arc, Hall effect in, *Child*, 370.
Electrodeless discharge in nitrogen, spectrum of, *Lewis*, 124.
Electrolytic rectifier, theory of, *Cook*, 23.
Electrolytes, molecular freezing point of water produced by concentrated solutions of, *Jones and Getman*, 146.
Emanation, radium, heating effect of, *Rutherford and Barnes*, 118.

F.

Fibers resembling fused quartz in their elastic properties, *Guthe*, 256.
Films, liquid, conductivity of, *Briggs and McLane*, 123.
Fluorescence, spectro-photometric study of, *Nichols and Merritt*, 122.
Fluorescent Solutions, Spectrophotometric Study of, Nichols and Merritt, 403.
Fluorescent Solutions, Effect of Light upon and the Electrical Conductivity of, Nichols and Merritt, 447.
Foley, A. L., Use of nickel in core of Marconi magnetic detector, 349.
Freezing point, molecular, of water produced by concentrated solutions of electrolytes, *Jones and Getman*, 146.
Frisbie, Fannie C., The Effect of Pressure on Magnetic Induction, 432.

G.

Ganz, A. F., *Ryan, Norris*, and *Hoxie's Electrical Machinery*, 382.
Gases, rarefied, conductivity produced in, by an incandescent kathode, *Merritt and Stewart*, 239.
Gates, Fanny C., Radiation from sulphate of quinine, 135.
Getman, F. H., Molecular lowering of freezing point of water produced by concentrated solutions of electrolytes, 146.
Gill, A. C., *Miers' Mineralogy*, 63.
Gonzenbach, E., *Interurban Electric Railways*, 459.
Gotschall, W. C., *Electric Railway Economics*, 459.
Gray, A. W., Convenient method of mounting carbon resistances, 57
Guthe, K. E., Fibers resembling fused quartz in their elastic properties, 256.
The Silver Voltameter, 445.

H.

Hall effect in the electric arc, *Child*, 370.
Hart, J. H., Continuous method of steam calorimetry, 263.
Heating effect of radium emanation, *Rutherford and Barnes*, 118.
Howland, H. N., *Elements of Physics*, 379.
Hoxie, G. L., *Electrical Machinery*, 382.

I.

Indices, refractive, by means of channeled spectra, *Williams*, 280.
Induction, mutual, determination of coefficients of, *Trowbridge*, 184.

J.

Jones, H. C., Molecular lowering of freezing point of water produced by concentrated solutions of electrolytes, 146.

K.

Kraus, C. A., Electrical conductivity of solutions in methyl alcohol in the neighborhood of their critical point, 40, 89.

L.

La Tour, H. B. de, The Induction Motor, 458.

Lewis, Percival, Spectrum of the afterglow in nitrogen at low pressure, 125.

Spectrum of electrodeless discharge in nitrogen, 124.

Light, velocity of, effect of intensity upon, *Doubt*, 129.

Light, velocity of, in a magnetic field, *Mills*, 65.

Lory, C. A., Differential telephone, 275.

Lyman, Theodore, Preliminary measurements of short wave-lengths, 320.

M.

Magie, Wm. F., The Volumes of Solutions, 449.

Magnetic detector, use of nickel in core of, *Foley*, 349.

Magnetic Field, Energy Density, Tension and Pressure in, *Barnett*, 452.

Magnetic Induction, Effect of Pressure on, *Frisbie*, 432.

Mann, C. R., *Edser's Light for Students*, 192.

Magnetic field, velocity of light in, *Mills*, 65.

McLane, J. W., Conductivity of liquid films, 123.

Mercury vapor, conduction in, *Wills*, 319.

Merritt, Ernest, Phosphorescence of organic substances at low temperatures, 120, 355.

Conductivity produced in rarefied gases by an incandescent kathode, 239.

Spectro-photometric study of fluorescence, 122.

A Spectrophotometric Study of Fluorescent Solutions belonging to Lommel's First Class, 403.

The Effect of Light Upon the Absorption and the Electrical Conductivity of Fluorescent Solutions, 447.

Metallic reflection of selenium, *Edmunds*, 193.

Miers, H. A., *Mineralogy*, 63.

Mills, John, Velocity of light in a magnetic field, 65.

N.

Nichols, E. F., *Clerke's Astrophysical Problems*, 378.

Nichols, E. L., A Spectrophotometric Study of Fluorescent Solutions belonging to Lommel's First Class, 403.

A Spectro-photometric study of fluorescence, 122.

The Effect of Light Upon the Absorption and the Electrical Conductivity of Fluorescent Solutions, 447.

Phosphorescence of organic substances at low temperatures, 120, 355.

Wade's Secondary Batteries, 384.

Nickel, use of, in core of Marconi Magnetic detector, *Foley*, 349.

Nitrogen at low pressures, spectrum of afterglow in, *Lewis*, 125.

Nitrogen, spectrum of electrodeless discharge in, *Lewis*, 124.

Norris, H. J., *Electrical Machinery*, 382.

P.

Perrine, F. A. C., *Conductors for Electrical Distribution*, 458.

Pendulum, compound, *Tatnall*, 187.

Phosphorescence of organic substances at low temperatures, *Nichols and Merritt*, 120, 355.

Physical Society, abstracts, 116, 188, 295, 370, 444.

Physical Society, minutes, 116, 295, 444.

Polarization recovery of cadmium cells, *Barnett*, 104.

Polarizer and compensator, half shade elliptical, *Brace*, 70.

Potential phenomena in vacuum tubes during the production and interruption of electrical discharge, *Taylor*, 321.

R.

Radio-activity, dependence upon concentration, *Rutherford*, 117.

Radio-activity of ordinary metals, *Burton*, 188.

Radiations, emitted by Righi vibrator, *Willard and Woodman*, 1.

Radiation from sulphate of quinine, *Gates*, 135.

Radiation produced in an alternating condenser field, *Sanford*, 366.

Radium emanation, heating effect of, *Rutherford and Barnes*, 118.

Rectifier, electrolytic, theory of, *Cook*, 23.

Reflecting Power of Selenium, *Edmunds*, 385.

Refractive indices by means of a channeled spectra, *Williams*, 280.

Resistance Boxes, Direct Reading Apparatus for Calibrating, *Wolff*, 456.

Resistance Standards, The Peculiar Behavior of, *Wolff*, 457.

Resistances, carbon, convenient method of mounting, *Gray*, 57.

Resonance in aërial systems, *Taylor*, 230.

Reynolds, F. G., The Viscosity Coefficient of Air with an Inquiry into the Effect of the Röntgen Rays thereon, 419.

Righi vibrator, radiations emitted by, *Willard and Woodman*, 1.

Röntgen Rays, Viscosity Coefficient of air with an Inquiry into the Effect of, *Reynolds*, 419.

Rutherford, E., Heating effect of radium emanation, 118.

Radio-activity, dependence upon concentration, 117.

Ryan, H. J., *Electrical Machinery*, 382.

Q.

Quinine, sulphate of, radiations from, *Gates*, 135.

S.

Sanford, Fernando, Radiation produced in an alternating condenser field, 366.

Saunders, F. A., Some additions to the Arc Spectra of the Alkali Metals, 452.

Selenium, metallic reflection of, *Edmunds*, 193.

Selenium, The Reflecting Power of, *Edmunds*, 385.

Solutions, The Volumes of, *Magie*, 449.

Spectra, channeled, refractive indices by means of, *Williams*, 280.

Spectrum of afterglow in nitrogen at low pressures, *Lewis*, 125.

Spectrum of electrodeless discharge in nitrogen, *Lewis*, 124.

Spectro-photometric study of fluorescence, *Nichols and Merritt*, 122, 403.

Steam calorimetry, continuous method of, *Hart*, 263.

Stewart, O. M., Conductivity produced in rarefied gases by an incandescent cathode, 239.

Stokes, George Gabriel, 58.

Swinburne, James, *Entropy*, 380.

T.

Taylor, A. H., Resonance in aërial systems, 230.

Taylor, S. N., Potential phenomena in vacuum tubes during production and interruption of electrical discharge, 321.

Tatnall, R. R., Theory of the compound pendulum, 187.

Telephone, differential, *Duane and Lory*, 275.

Thomson, *Applied Mechanics*, 375.

Timmerman, C. E., *Laboratory Manual of Physics*, 64.

Trevor, J. E., *Voigt's Thermodynamik*, 372.

Trowbridge, A., Determination of coefficients of mutual induction, 184.

V.

Vacuum tubes, potential phenomena in, during production and interruption of electrical discharge, *Taylor*, 321.

Velocity of light, effect of intensity upon, *Doubt*, 129.

Velocity of light in a magnetic field, *Mills*, 65.

Vibrator, Righi, radiations emitted by, *Willard and Woodman*, 1.

Voigt, W., *Thermodynamik*, 372.

Voltameter, A Study of the Silver, Guthe, 445.

W.

Wade, E. J., *Secondary Batteries*, 384.
Water, freezing point of, produced by concentrated solutions of electrolytes, *Jones and Getman*, 146.
Wave lengths, short, preliminary measurements of, *Lyman*, 320.
Webster, A. G., Presidential address, 297.
Willard, H. R., Radiations emitted by Righi vibrator, 1.
Williams, S. R., Determination of refractive indices by means of channeled spectra, 280.
Wills, A. P., Conduction in mercury vapor, 319.
Wolff, F. A., The Peculiar Behavior of Some Resistance Standards and its Explanation, 457.
Wolff, F. A., A Direct Reading Apparatus for Calibrating Resistance Boxes, 456.
Wolff, F. A., The Standard Cell, 454.
Woodman, L. E., Radiations emitted by Righi vibrator, 1.

JAN 19 1917